

Lesson 47: Prisms and Cylinders

**Weekly Focus:** prisms, cylinders  
**Weekly Skill:** calculate area and volume

**LESSON 47: Prisms and Cylinders**

**Lesson Summary:** For the warm up, students will solve a problem about the earth and the moon. In Activity 1, students will find the surface area and volume of a prism. In Activity 2, they will do the same for a cylinder. Activity 3 is a worksheet to practice the computations. Activity 4 is the word problems in the student book and workbook. Activity 5 is a real-life application question. There is also an optional extra activity for homework. Estimated time for the lesson is 2 hours.

**Materials Needed for Lesson 47:**

- Boxes, cylinders (cans or oatmeal containers are good), rulers, scissors, and paper.
- Notes on volume and surface area of cylinders and prisms. The geometry notes come from this site: <http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf> (pages 18-22)
- Video A (length 3:50) on volume of a prism.  
Video B (length 10:30) on finding surface area of prisms.  
Video C (length 5:20) on surface area and volume of cylinders.  
The videos are required for teachers and recommended for students.
- 1 Worksheet (47.1) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Student Book (pages 106 – 107)*
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook (pages 150 – 153)*
- 1 Application Activity (attached)
- 1 Extra Activity for Homework or Extra Time (attached)

**Objectives:** Students will be able to:

- Solve the earth and moon dimensions word problem
- Find the volume and surface areas of a cylinder (can) and a prism (box)
- Calculate the volume and surface areas and solve word problems
- Do a real-life application activity about pouring a concrete driveway.

**ACES Skills Addressed:** N, CT, LS, ALS

**CCRS Mathematical Practices Addressed:** Building Solution Pathways, Mathematical Fluency, Model with Math

**Levels of Knowing Math Addressed:** Intuitive, Pictorial, Concrete, Abstract, and Application

**Notes:**

**You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.**

**For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The “easier” problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.**

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

Lesson 47: Prisms and Cylinders

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

**Lesson 47 Warm-up: Solve the earth and moon problem**

**Time: 10 Minutes**

Write on the board: The circumference of the earth is 40,075 km. The circumference of the moon is about 27% that of the earth.

Basic Questions:

- What is the circumference of the moon in km?
  - $(0.27)(40,075) = 10,820$  km
- What is the circumference of the moon in miles?
  - Hint if students need it:  $0.6 \times \text{km} = \text{miles}$
  - $(0.6)(10,820) = 6,492$  miles. *C of moon is about 6,500 miles.*
- What is the circumference of the earth in miles?
  - $(0.6)(40,075) = 24,045$  miles

Extension Questions:

- If the earth's diameter is 12,750 km, what is its radius?
  - *Half of the diameter is the radius = 6,375 km*
- If the moon's radius is 1738 km, what is its diameter?
  - *Radius  $\times 2 =$  diameter = 3,476 km*
- Note to teacher: The above are all estimates. If you look up the dimensions of the earth and the moon online, you will find more precise measurements.

**Lesson 47 Activity 1: Volume and Surface Area of Prisms**

**Time: 15 Minutes**

1. The objective of Activity 1 is to learn how to find the volume and the surface area of a prism.
2. A **prism is a 3-dimensional figure with 2 congruent bases**. A good example is a box; the two ends are the bases.
3. Give students boxes and rulers. Have students measure the area of each surface. They will realize they only need to measure the area of the 3 different sides and double those to get the surface area of the whole box.
4. If you have time or someone needs more of a visual, have the students cut out sheets of paper that fit each of the 6 surfaces. The total surface area is the area of the top and bottom, the two ends (bases), and the two sides all combined.
5. Students have discovered the formula for surface area, which is the same as in the **Lesson 47 Notes, pages 18 – 22**, where the surface area formula is given as  $2(lw + lh + wh)$ .

Lesson 47: Prisms and Cylinders

6. The surface area as given on the formula sheet of the GED test is shorter. It is **SA = ph + 2B** where p=perimeter of a base, h= height, and B= area of the base.
7. Students can use the shorter formula once they understand the meaning of surface area.
8. Notes:
  - a. As an alternative (if short on time), give the students the formula without taking time to have them discover it for themselves.
  - b. The video and worksheet below refer to the **lateral area**, which is  $ph$  = perimeter of the base x height of prism
  - c. Remind students that area is always squared.
  - d. Use the **Notes, pages 18 – 22**, as a teaching guide or copy them for the students if they are helpful.
9. Now students can measure the volume of the box. The **volume** is how much space is occupied by an object.
10. Use **B**, the area of the base (one of the ends), and multiply it by the height.
11. Use Example 1 in the notes as another practice example. Give the students the measurements and have them use their GED formula sheet (a portion of it is given below) to solve for the surface area and the volume of the box.

**Surface area and volume of a:**

rectangular/right prism	$SA = ph + 2B$	$V = Bh$
cylinder	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
pyramid	$SA = \frac{1}{2}ps + B$	$V = \frac{1}{3}Bh$
cone	$SA = \pi rs + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

(p = perimeter of base with area B;  $\pi \approx 3.14$ )

**Lesson 47 Activity 2: Volume and Surface Area of Cylinders** | Time: 15 Minutes

1. Give students a can or other type of cylinder, paper, ruler, and scissors.
2. Have them find the area of the top or bottom (a circle). They may remember the formula of a circle as  $\pi r^2$ .
3. To find the surface area of the outside of the can, have them cut out a piece of paper that will wrap around the can and then measure its area (width x length). This area around the can is called the lateral area, as described in the video.
4. The area of that sheet of paper should measure about the same as the lateral area given in the formula:  **$2\pi r h$**
5. The formula for SA of a cylinder is the area of two bases (top and bottom) of cans added to the lateral area. The formula is  **$2\pi r h + \pi r^2$** .
6. The **volume** of a cylinder is measured as  **$\pi r^2 h$** , which is the area of the base times the height.
7. Have students practice measuring volume with the cans or cylinders they have.
8. Have students do example 2 from the notes as another practice.

Lesson 47: Prisms and Cylinders

**Lesson 47 Activity 3: Practice Calculations**

**Time: 15 Minutes**

1. Hand out **Worksheet 47.1**.
2. Questions 1 to 6 are to practice labeling the different parts of the prisms or cylinders.
3. Questions 7 to 12 are to solve for surface area.

**Lesson 47 Activity 4: Word Problems**

**Time: 45 Minutes**

1. Do the problems in the **student book pages 106-107** together.
2. Have students work independently in the **workbook pages 150 to 153**.
3. Do any of the challenging problems on the board if there is time.

**Lesson 47 Application A: Pouring a Concrete Driveway**

**Time: 20 Minutes**

Give students the problem:

Mike Jones bought an older house and wants to put in a new concrete driveway. The driveway will be 30 feet long, 10 feet wide, and 9 inches thick. Concrete is measured by the cubic yard. One sack of dry cement mix costs \$7.30, and it takes four sacks to mix up one cubic yard. How much will it cost Mike?

See the attached explanation and answer.

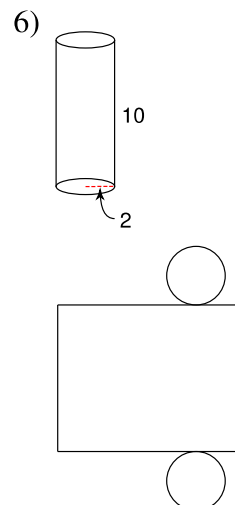
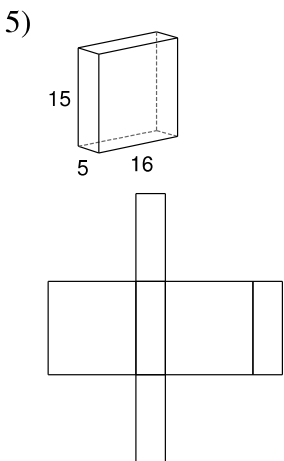
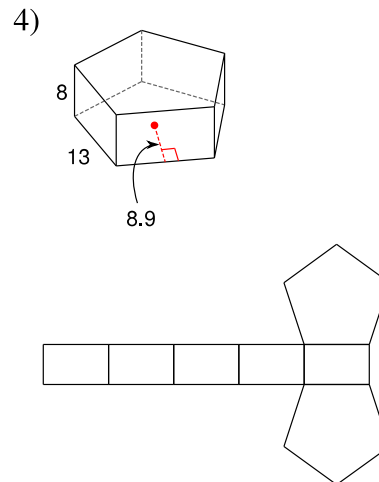
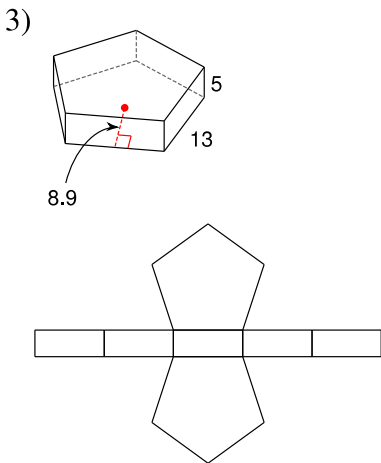
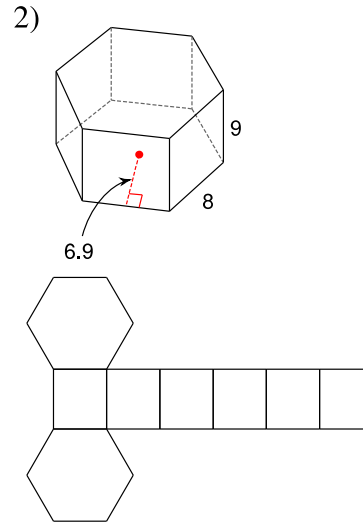
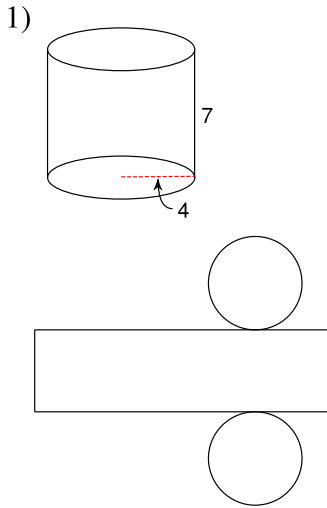
**Extra Time? Finish Early? Make a Box**

**Time: 20 Minutes**

1. Give students the directions for this extra activity or for homework.
2. It is helpful for those who learn kinesthetically by doing something.

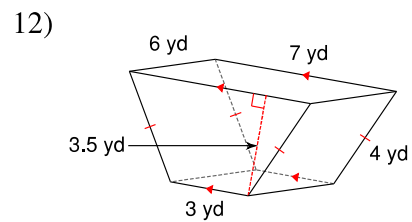
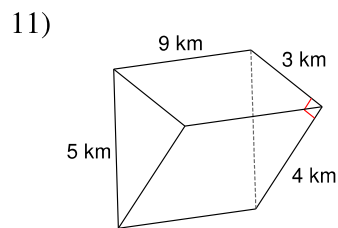
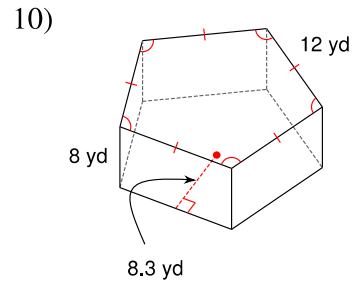
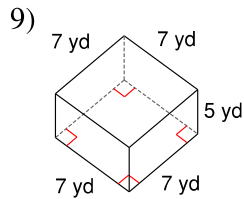
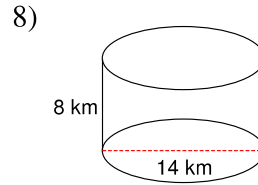
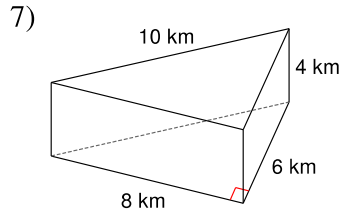
Lesson 47: Prisms and Cylinders

Worksheet 47.1



Lesson 47: Prisms and Cylinders

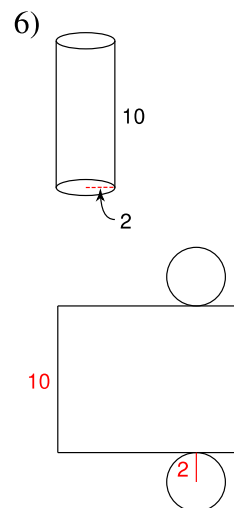
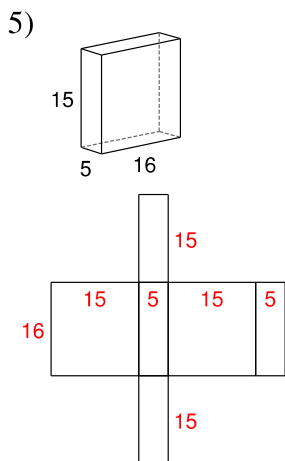
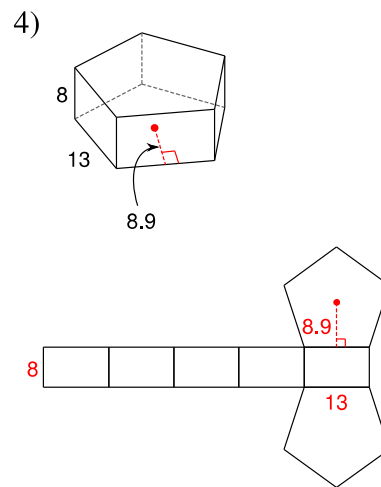
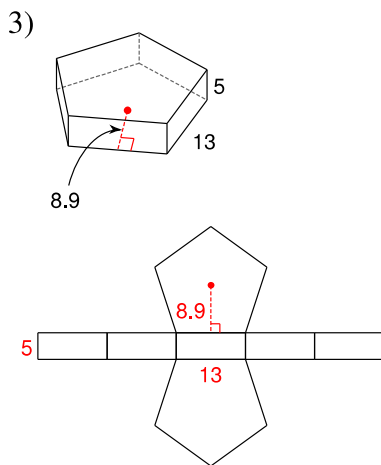
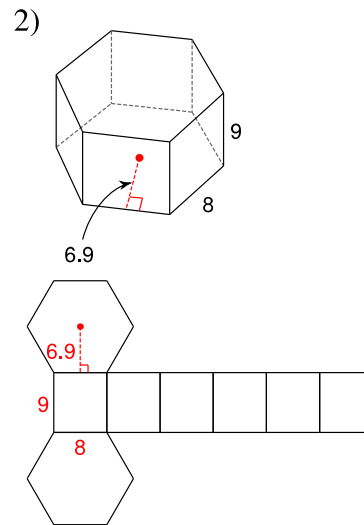
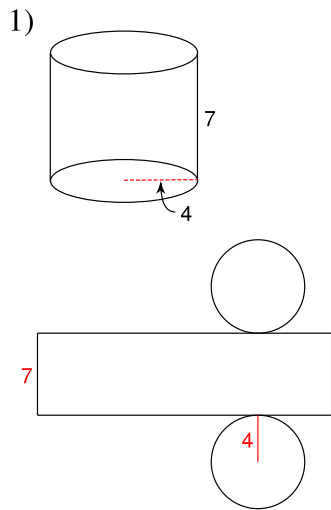
Find the lateral area and surface area of each figure. Round your answers to the nearest thousandth, if necessary.



Lesson 47: Prisms and Cylinders

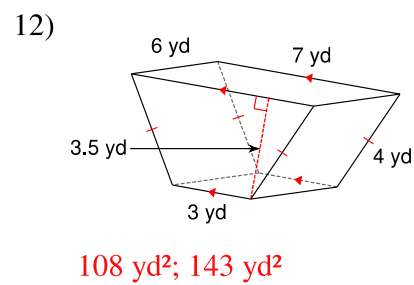
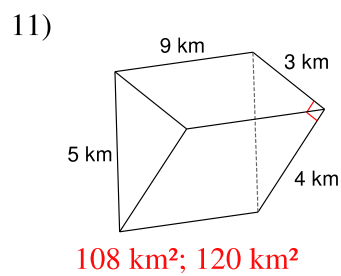
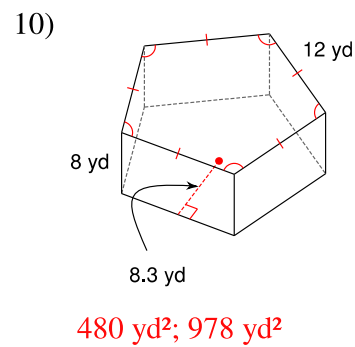
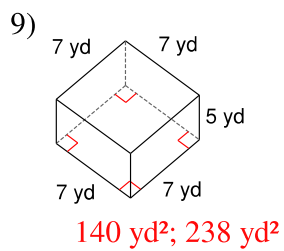
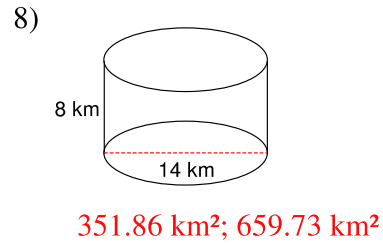
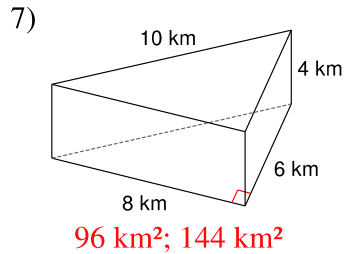
Worksheet 47.1 **Answers**

Copy the measurements given onto the net of each solid.



Lesson 47: Prisms and Cylinders

Find the lateral area and surface area of each figure. Round your answers to the nearest thousandth, if necessary.



### Lesson 47 Application: Pouring a Concrete Driveway

Mike Jones bought an older house and wants to put in a new concrete driveway. The driveway will be 30 feet long, 10 feet wide, and 9 inches thick. Concrete (a mixture of sand, gravel, and cement) is measured by the cubic yard. One sack of dry cement mix costs \$7.30, and it takes

four sacks to mix up 1 cubic yard of concrete. How much will it cost Mike to buy the cement?

**Solution:**

The driveway that is being poured will be a rectangular solid or box. Thus in order to answer this question, we will first need to find the volume of this box (the amount of cubic units it will take to fill this box). The problem tells us that concrete is measured by the cubic yard. This lets us know that those are the units we will want to calculate with. None of the dimensions of the driveway are given in yards. We will need to use our dimensional analysis (unit conversion) to convert all of the measurements to yards. This could be done at a later point, but this is the easiest place to take care of the conversion.

Convert length:

$$\frac{30 \text{ feet}}{1} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 10 \text{ yards}$$

Convert width:

$$\frac{10 \text{ feet}}{1} \times \frac{1 \text{ yard}}{3 \text{ feet}} = \frac{10}{3} \text{ yards}$$

We are not going to estimate this

Lesson 47: Prisms and Cylinders

value since the approximation with introduce error. As in the finance section, we don't want to round until the end of the problem or in a place where it is absolutely necessary.

Convert height:

$$\frac{9 \text{ inches}}{1} \times \frac{1 \text{ yard}}{36 \text{ inches}} = \frac{1}{4} \text{ yards} = .25 \text{ yards}$$

Since the decimal representation of this number is a terminating decimal, we can use this representation in our calculations.

Volume of driveway:

We are now ready to calculate the volume of the driveway.

$$V = lwh = (10) \left( \frac{10}{3} \right) (.25) = \frac{25}{3} \text{ cubic yards.}$$

Now, we are told that it takes 4 bags of cement to make one cubic yard of concrete. So we will now calculate how many bags of cement to buy. Mike's driveway is  $\frac{25}{3}$  cubic yards and each of those cubic yards requires 4 bags of cement. Thus Mike will need  $\frac{25}{3} \times 4 = \frac{100}{3}$  bags or approximately 33.333 bags. Here is where common sense needs to come in. There is no store that will sell .333 bags of cement mix. Stores only sell whole bags of cement mix. Thus we will need to round up to the next whole bag (we can't round down or we will not have enough cement mix to complete the driveway). This means that Mike will need to buy 34 bags of cement mix. Each of these bags will cost \$7.30. This means that Mike will pay  $7.30 \times 34 = \$248.20$  for the cement mix for this driveway.

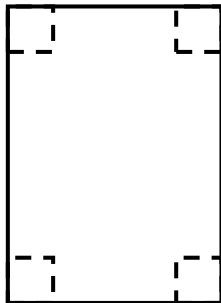
**Activity: Extra Time? Finish Early?**

**Example 8:**

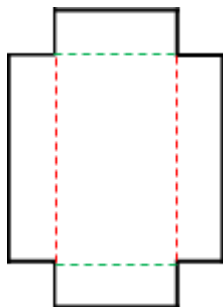
From an 8.5-inch by 11-inch piece of cardboard, 2-inch square corners are cut out and the resulting flaps are folded up to form an open box. Find the volume and surface area of the box.

**Solution:**

For this problem, it will be really helpful to make the box described above. You start with a standard piece of paper. You cut out the dashed square indicated below from each corner. You make sure each side of the square is 2 inches in length.



What you are left with is the shape below.



You now fold up along the dashed lines to create a box. The box that we have created is a rectangular solid. This box has no top. Not having a top will not affect the volume of the box.

We only need to determine the length of the base of the box, the width of the base of the box, and the height of the box. The red dashed lines represent the length of the base of the box. The original length of the paper was 11 inches. We removed 2 inches from the top of the page and we also removed 2 inches from the

Lesson 47: Prisms and Cylinders

bottom of the page. Thus the red dashed line is  $11 - 2 - 2 = 7$  inches.

The green dashed line represents the width of the base of the box. The original width of the paper was 8.5 inches. We removed 2 inches from the left side of the page and also removed 2 inches from the right side of the page. Thus the green dashed line is  $8.5 - 2 - 2 = 4.5$  inches.

We now need to think about the height of the box. Since we have folded up the sides for form the height of the box, we just need to determine how tall those sides are. Since they were made by cutting out 2-inch square from each corner, these sides must be 2 inches high.

Now we are ready of calculate the volumes  
 $V = lwh = (7)(4.5)(2) = 63$  cubic inches.

Now we need to calculate the surface area of our box. Since there is no top to this box, we can start formula for the surface area of a box. We will then need to subtract off the area of the top of the box. This will give us

$SA = 2(lw + wh + lh) = 2(7 \times 4.5 + 4.5 \times 2 + 7 \times 2) = 109$  square inches for the box with the top included. The top would have the same area as the base of the box. This would be  $A = lw = (7)(4.5) = 31.5$  square inches. Thus the surface area of our figure is total surface area - area of the top =  $109 - 31.5 = 77.5$  square inches.

There is another way to calculate the surface area of this box. The surface area is the amount of paper it would take to cover the box without overlap. You should notice that this is the same as the amount of paper we used to make the box. Thus, it is enough to calculate the area of the paper as shown here.

