Weekly Focus: functions
Weekly Skill: comparison and application

LESSON 39: Comparison of Functions

Lesson Summary: For the warm-up, students will solve a problem about hiking. Activity 1 is to help students compare functions that are in table vs. graph format. Activity 2 includes functions in various forms with related questions. Activity 3 consists of examples of what happens when graphs are manipulated in different ways. Activity 4 provides more challenging practice problems in the student book and workbook. Activity 5 is an application of functions in which students compare different cell phone plans, Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 39:
- Video (length 5:15) on comparison of functions. The video is required for teachers, optional for students.
- Notes 39A sheet (attached)
- 3 Worksheets (39.1, 39.2, and 39.3) with answers (attached)
    - Worksheet 39.3 is to be used for examples for Activity 3.
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Student Book (pages 82 – 83)*
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook (pages 122 – 125)*
- Application Activity (link embedded in lesson plan)
- Notes:
    - Advise students that this is the last new lesson in the algebra unit. The next lesson will be an algebra test.

Objectives: Students will be able to:

- Solve the word problem about hiking, time, and distance as well as graph it
- Compare functions represented as tables, graphs, equations, and/or verbal statements
- Apply their skills by making tables and graphs of different cell phone plans

ACES Skills Addressed: N, CT, LS, EC
CCRS Mathematical Practices Addressed: Building Solution Pathways, Mathematical Fluency, Reason Abstractly and Quantitatively, Look For and Make Use of Structure
Levels of Knowing Math Addressed: Intuitive, Pictorial, Abstract, and Application

<u>Notes:</u>
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

| Lesson 39 Warm-up: Solve the hiking problem	Time: 5 Minutes

<u>Write on the board:</u> Courtney hikes up a mountain peak at a speed of 2.5 miles per hour and it takes her 3 hours to reach the top. She rests for a half hour and takes 1.75 hours to go back down the same route.

<u>Basic Questions:</u>
- How many miles has she hiked total? *(2.5 mi. x 3 hours = 7.5 mi. each way = 15 miles total)*
- If she starts at 8:00 a.m., at what time is done hiking? *(3 hours up + 30 min. rest + 1hr. 45 min. down = 5 hr. 15 min. of hiking so she is done at 1:15 pm)*

<u>Extension Question:</u>
- Draw a line graph of Courtney's hike. *Students' graphs should look similar to:*
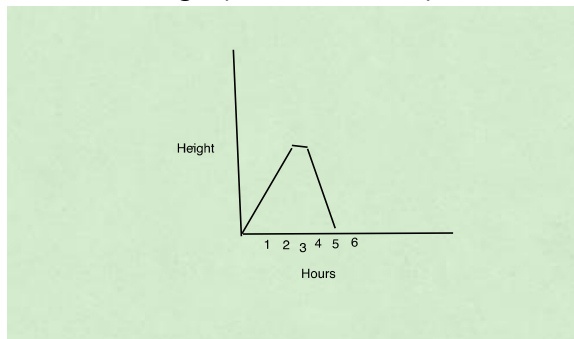
Lesson 39 Activity 1: Comparing Functions in Different Formats	Time: 20 Minutes

1. Review the meaning of functions and their different representations:
   a. Each input x has exactly one output f(x).
   b. Functions can be represented with tables, equations, graphs and/or verbal statements.
   c. They can be compared via their slopes, intercepts, minimums and maximums.
   d. You can compare 2 linear functions, 2 quadratic functions, or one of each.
2. Use the attached **Notes 39A Guided Lesson** for examples and explanations.

3. Note for teacher:
    a. The notes may be copied for students or projected on the wall.
    b. It may be helpful to graph each of the functions listed in table format. This will help deepen students' understanding as well as practice more graphing.
4. Do **Worksheet 39.1** for practice.

Lesson 39 Activity 2: Comparing Functions by Graphing	Time: 10 Minutes

1. In this activity, students will practice graphing functions that are represented verbally or answering questions about a graph.
2. Do **Worksheet 39.2.**

Lesson 39 Activity 3: Manipulating the Graphs of Functions	Time: 20 Minutes

1. Students have experience with looking at functions and their representations from previous lessons and from Activities 1 and 2.
2. Copy **Worksheet 39.3** for the students and use the questions as examples to practice what happens when a function is manipulated.
3. Note: The worksheet has "independent practice" in the title but you won't use it as such.
4. Do the 9 questions together as necessary. If there is time, have students graph the functions to help make it easier to understand the functions.

Lesson 39 Activity 4: Comparison of Functions Practice	Time: 45 Minutes

1. Review the meanings of slope and intercepts and the different ways to find them as explained in the left column on page 82 in the student book.
2. Do **pages 82 – 83** in the **student book**.
3. Do the **workbook pages 122 – 125** for additional practice.
4. Note: These problems may be challenging for students and you may need to do many of them together as a class. If you do, still give students some time to work alone to try to figure them out on their own.

Lesson 39 Application: Comparing iPhone Plans	**Time: 20-30 Minutes**

1. The application activity can serve as a culminating activity of graphing and comparing functions. The solution can be accessed if you are a member.

2. Students can do it for homework if there isn't enough time in class.

3. It can, however, foster a rich discussion with critical thinking if done in class.

Lesson 39: Comparison of Functions

Notes 39A

Comparing Functions in Different Formats – Guided Lesson

Complete the following problems:

1) Two functions are represented in different ways.

   Function 1: The input-output table shows the x- and y-values of a quadratic function.

x	y
0	0
2	3
4	15
6	35
8	63
10	99
12	143

   Function 2: The graph of a linear function is shown.
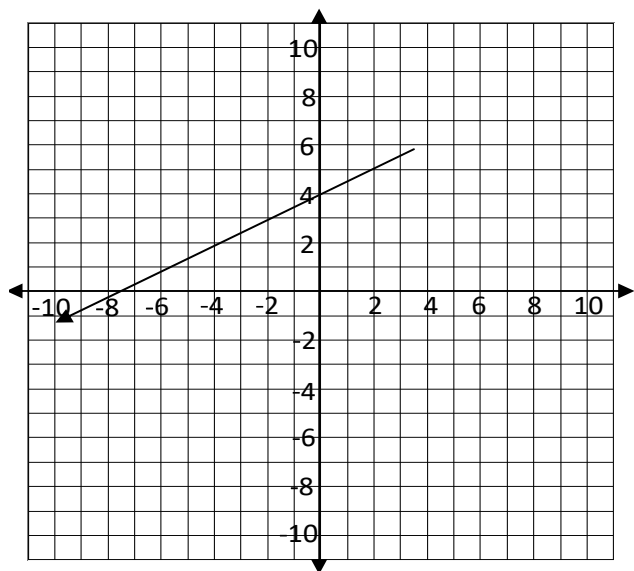
   From the two functions, which function grows faster for large positive values of x?

Comparing Functions in Different Formats – Guided Lesson Explanation

For all of these problems we are looking to compare the growth or lack of growth of the data. Growth of data can be gauged by the slope of the line created by the data. In a sense, slope indicates the rate of change.

The steeper the slope of the line created by the data, the more growth. If you are looking for lack of growth in the data, you are looking for shallow rise in the slope, when compared to another set of data. If the slope is negative, there is no growth.

Slope intercept form is defined as y = mx + b

m = slope   b = y-intercept (where the line crosses the y-axis)

You can also determine slope using the following equation when x_1 is not equal to x_2.

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

Explanation #1
Step 1) Find two points where x_1 is not equal to x_2 for both functions.

Function 1: I chose points (2, 3) and (6, 35)
Function 2: I chose points (-2, 3) and (2, 5)

Step 2) Determine the slope (m) for both functions based off of the points that were chosen.

Function 1: $\dfrac{35 - 3}{6 - 2} = \dfrac{32}{4} = 8$

Function 2: $\dfrac{5 - 3}{2 - -2} = \dfrac{2}{4} = 0.5$

Step 3) Interpret the difference in slope between the functions.

Function 1's slope is much greater. This indicates that Function 1 grows faster for large positive values of x.

Worksheet 39.1 Practice Comparing Functions in Different Formats

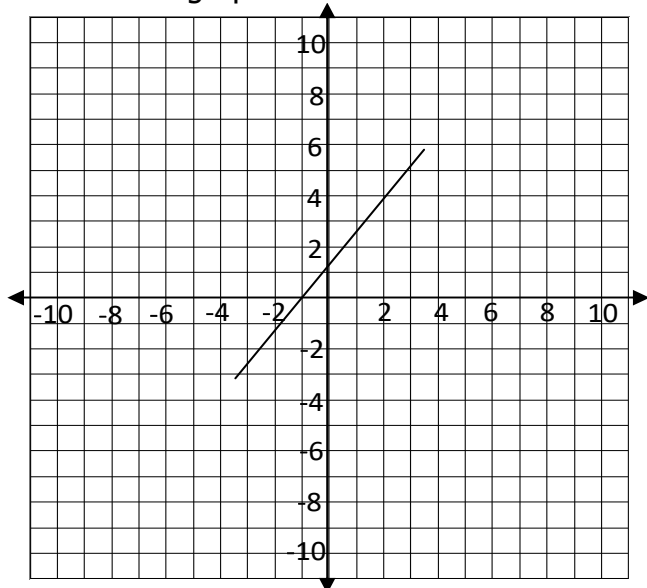| Comparing Functions in Different Formats – Independent Practice Worksheet |
|---|

Complete all the problems.

1) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y- values of a quadratic function.

x	y
1	1
3	9
4	16
5	25
6	36
7	49
8	64

Function 2: The graph of a linear function is shown.

From the two functions, which function grows faster for large positive values of x?

2) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y- values of a quadratic function.

x	y
0	0
1	1
2	4
4	16
6	36
7	49
10	100

Function 2: The graph of a linear function is shown.
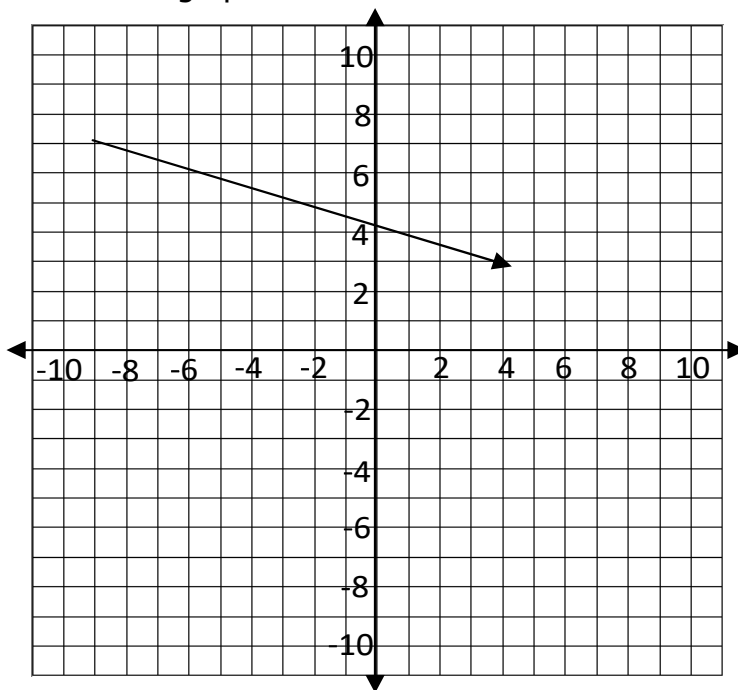
Which of the two functions shows no growth at all?

3) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y- values of a quadratic function.

x	y
2	7
4	19
6	39
8	67
10	103
12	147
13	172

Function 2: The graph of a linear function is shown.
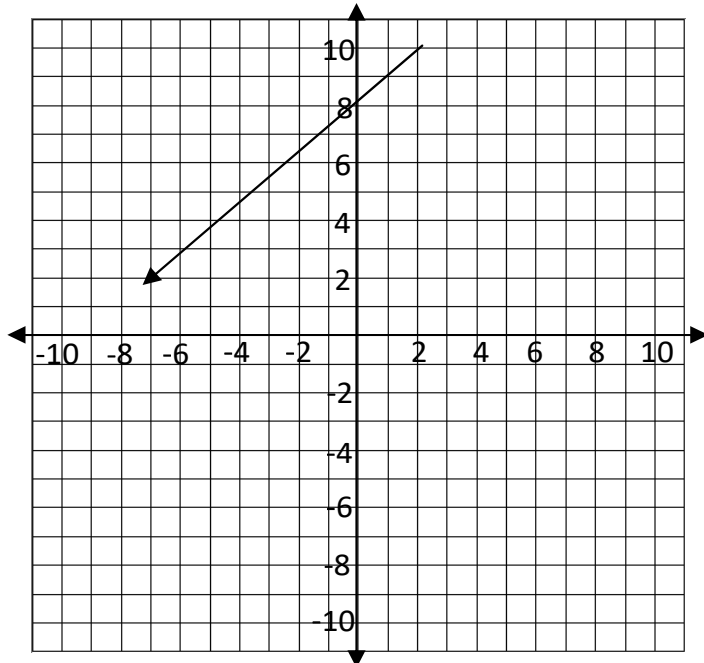
Which of the two functions grows at a faster rate?

Worksheet 39.1 Answers

(1) Function 1
(2) Function 2
(3) Function 1

Worksheet 39.2 Compare Functions by Graphing

Analyzing Functional Relationships by Graphing - Independent Practice

Complete all the problems.

1. Michael is going to the playground. First, he walks to church. He stays there for few minutes. Then he hires a taxi to drive him to the playground. The taxi stops at a coffee shop and then the playground. Draw the graph representing Michael's trip to the playground.

2. Describe the graph of the function. Is this linear or non-linear?

x	1	2	3	4	5
y	5	10	15	20	25

3. Describe the graph of the function between y = 2 and y = 4.

4. Describe the graph of the function. Is this relationship non-linear?

5. Isabella is a plumber. She walks to the market. She buys some tools and stays there a few minutes. Then she hires a taxi to go to Sarah's house. After Sarah's she takes the taxi to Olivia's house. Draw the graph representing Isabella's trip to Olivia's house.
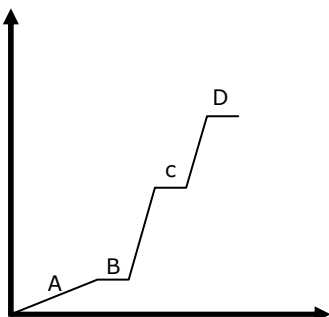
Worksheet 39.3 Manipulating the Graphs of Functions

Manipulating the Graphs of Functions - Independent Practice Worksheet

Complete all the problems.

1. The function y = 11x - 8, which statement best describes the effect of increasing the y-intercept by 7?

a. The new line is parallel to the original.

b. The new line has greater rate of change.

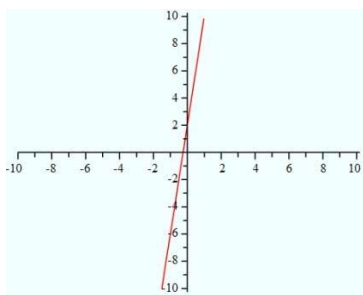2. Which statement best describes the effect on the graph of f(x) = 6x - 5

If the y intercept is changed to + 12?

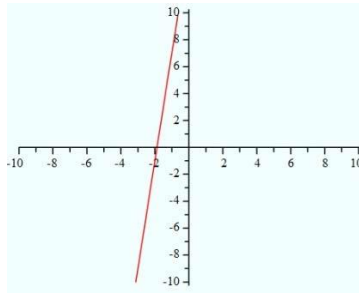a. The slope does not change.

b. The new line passes through the origin.

3. The graph of a line that contains the points (-7, -5) and (2, 4) is shown below.

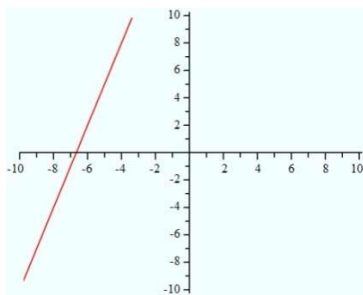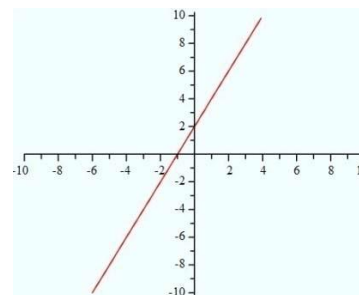Which best represents this line if the slope is doubled and the y-intercept remains constant?

a)

b)

c)

d)

4. The function y = 3x - 5, which statement best describes the effect of increasing the y-intercept by 7?

a. The new line is parallel to the original.

b. The new line has greater rate of change.

5. Which statement best describes the effect on the graph of f(x) = 8x - 3

If the y intercept is changed to + 5?

a. The slope remains constant.

b. The new line passes through the origin.

6. The function y = 4x - 6, which statement best describes the effect of increasing the y-intercept by 7?

a. The new line is parallel to the original.

b. The new line has greater rate of change.

7. Which statement best describes the effect on the graph of f(x) = 14x - 4

If the y intercept is changed to + 6?

a. The slope stays the same.

b. The new line passes through the origin.

8. Which statement best describes the effect on the graph of f(x) = 5x - 8

If the y intercept is changed to + 2?

a. The slope does not change.

b. The new line passes through the origin.

9. The function y = 10x - 12, which statement best describes the effect of increasing the y-intercept by 4?

a. The new line is parallel to the original.

b. The new line has greater rate of change.

Lesson 39: Comparison of Functions

Worksheet 39.3 Manipulating the Graphs of Functions Answers

Manipulating the Graphs of Functions - Independent Practice Worksheet Answer Key

1. a. Yes, the new line is parallel to the original.

2. a. Both lines are parallel, so the slope is the same.

3. d.
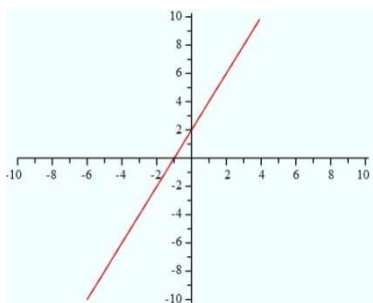
4. a. Yes, the new line is parallel to the original.

5. a. Both lines are parallel, so the slope is the same.

6. a. Yes, the new line is parallel to the original.

7. a. Both lines are parallel, so the slope is the same.

8. a. Both lines are parallel, so the slope is the same.

9. a. Yes, the new line is parallel to the original.