

Lesson 37: Graphing Quadratic Equations

LESSON 37: Graphing Quadratic Equations

Weekly Focus: quadratic equations
Weekly Skill: graphing

Lesson Summary: For the warm-up, students will solve a problem about mean, median, and mode. In Activity 1, they will learn the basics of graphing quadratic equations (parabolas). In Activities 2 and 3, students will answer problems in the student book and the workbook. As an alternative or prior to the student book problems, you may want to assign Activity 5, which is graphing of parabolas. Activity 4 is an application activity about throwing snow up into a pile. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 37:

- Video (length 10:53) on graphing parabolas. The video is required for teachers and recommended for students.
- Notes 37A for the teacher and students
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Student Book (pages 78 – 79)*
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook (pages 114 – 117)*
- Application Activity (link embedded in the lesson plan)
- 1 Worksheet (37.1) with answers (attached)

Objectives: Students will be able to:

- Understand the meaning of parabolas
- Solve problems about parabolas
- Graph parabolas

ACES Skills Addressed: N, CT, LS

CCRS Mathematical Practices Addressed: Model with Math, Reason Abstractly and Quantitatively

Levels of Knowing Math Addressed: Intuitive, Abstract, Pictorial and Application

Notes:

You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The “easier” problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

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This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 37 Warm-up: Solve the mean, median, and mode question

Time: 5 Minutes

Write on the board: Jonathan keeps track of his bowling scores and records the data. His results so far are: 221, 186, 171, 126, 208, and 186.

Basic Questions:

1. What is the mean of Jonathan's scores?
2. What is the median?
3. What is the mode?
4. What is the range?

Answers:

1. 183 (the mean is the average)
2. 186 (the median is the middle; if there are 2, average the 2)
3. 186 (the most is the most frequently occurring)
4. 95 (the range is the difference between the highest and lowest)

Extension Question:

1. How would the median change if Jonathan bowled one more game and scored 171?
a. It would not change because 186 would still be in the middle

Lesson 37 Activity 1: Graphing Quadratic Equations

Time: 15-20 Minutes

1. First, draw the basic parabola of $y = x^2$ on the board. (It should look similar to the one on page 4 of the notes).
2. Explain that this graph is a parabola, not a line, because it represents the graph of a basic quadratic equation.
3. When are parabolas used?
 - a. A parabola with a vertex as a minimum (such as the one here) is used for measuring reflection, for satellite dishes, for headlights on a car or to measure how high up a snow pile is (as we will see in the application activity) as examples.

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b. Parabolas that are upside down (with the vertex as the highest point) show the trajectory of a ball that is thrown up in the air or fireworks or a mortar in combat. Draw a quick example on the board for the students.

4. The most basic quadratic equation is $y = x^2$.
5. The lowest point on this parabola is (0,0). It has the **minimum value** and is called the **vertex**.
6. Make an in/out table to show some of the points on the graph. Some points may be: (-2,4), (-1,1) (0,0), (1,1), (2,4).
7. Now we will look at graphs of the standard form of quadratic equations: $ax^2 + bx + c = 0$.
8. Give students copies of the attached **Notes 37A**.
9. The main focus of the lesson is Section C: Graphs of quadratic equations are parabolas.
Explain #3: The movement of parabolas on the graph by making an in/out table of the example equations. This will help students see why the parabola moves up or down, left or right.
10. Students learned to factor quadratic equations in an earlier lesson but notes are included here for their review.

Lesson 37 Activity 2: The Meaning of Parabolas

Time: 15 Minutes

1. Do **pages 78-79 in the student book**.
2. Explain **maximum** when $a < 0$ and **minimum** when $a > 0$.
3. Also draw the **axis of symmetry**, which is a vertical line drawn through the vertex. The half of the parabola on one side of the axis is a reflection of the half on the other side.

Lesson 37 Activity 3: Independent Practice

Time: 30 Minutes

Have students work independently in the **workbook pages 114-117**. Circulate to help. Review any questions that students found challenging. Choose a few problems to have volunteer students do on the board and explain if they want.

Lesson 37 Activity 4 Application: Throwing Snow

Time: 25 Minutes

This activity is a good application activity for Minnesotans who know what snow piles are. You can [download the activity](#) directly from yummymath.com for free. The solution can be accessed if you are a member.

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Lesson 37 Activity 5: Graphing Practice or Homework

Worksheet 37.1 can be assigned as practice to graph quadratic equations *before* doing the activities in the student book and workbook or can be given for homework.

Notes 37A Quadratic Equations

A. Definition

1. A quadratic equation is an equation that does not graph into a straight line. The graph will be a smooth curve.
2. An equation is a quadratic equation if the highest exponent of the variable is 2. Some examples of quadratic equations are: $x^2 + 6x + 10 = 0$ and $6x^2 + 8x - 22 = 0$.
3. A quadratic equation can be written in the form: $ax^2 + bx + c = 0$. The a represents the coefficient (the number) in front of the x^2 variable. The b represents the coefficient in front of the x variable and c is the constant.
 - a. For example, in the equation $2x^2 + 3x + 5 = 0$, the a is 2, the b is 3, and the c is 5.
 - b. In the equation $4x^2 - 6x + 7 = 0$, the a is 4, the b is -6 , and the c is 7. In the equation $5x^2 + 7 = 0$, the a is 5, the b is 0, and the c is 7.
 - c. Is the equation $2x + 7 = 0$ a quadratic equation? No! The equation does not contain a variable with an exponent of 2. Therefore, it is not a quadratic equation.

B. Review of Solving Quadratic Equations Using Factoring

1. Why is the equation $x^2 = 4$ a quadratic equation? It is a quadratic equation because the variable has an exponent of 2.
2. To solve a quadratic equation:
 - a. First make one side of the equation zero. Let's work with $x^2 = 4$.
 - b. Subtract 4 from both sides of the equation to make one side of the equation zero: $x^2 - 4 = 4 - 4$.
 - c. Now, simplify $x^2 - 4 = 0$. The next step is to factor $x^2 - 4$.
 - d. It is factored as the difference of two squares: $(x - 2)(x + 2) = 0$.
 - e. If $ab = 0$, you know that either a or b or both factors have to be zero because a times $b = 0$. This is called the **zero product property**, and it says that if the product of two numbers is zero, then one or both of the numbers have to be zero. You can use this idea to help solve quadratic equations with the factoring method.
 - f. Use the zero product property, and set each factor equal to zero: $(x - 2) = 0$ and $(x + 2) = 0$.
 - g. When you use the zero product property, you get linear equations that you already know how to solve.

Solve the equation:	$x - 2 = 0$
Add 2 to both sides of the equation.	$x - 2 + 2 = 0 + 2$
Now, simplify:	$x = 2$
Solve the equation:	$x + 2 = 0$
Subtract 2 from both sides of the equation.	$x + 2 - 2 = 0 - 2$
Simplify:	$x = -2$

You got two values for x . The two solutions for x are 2 and -2 .

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All quadratic equations have two solutions. The exponent 2 in the equation tells you that the equation is quadratic, and it also tells you that you will have two answers.

Tip: When both your solutions are the same number, this is called a **double root**. You will get a double root when both factors are the same.

Before you can factor an expression, the expression must be arranged in descending order. An expression is in descending order when you start with the largest exponent and descend to the smallest, as shown in this example: $2x^2 + 5x + 6 = 0$.

Example B

$$x^2 - 3x - 4 = 0$$

Factor the trinomial $x^2 - 3x - 4$.

Set each factor equal to zero.

Solve the equation.

Add 4 to both sides of the equation.

Simplify.

Solve the equation.

Subtract 1 from both sides of the equation.

Simplify.

The two solutions for the quadratic equation are 4 and -1.

$$(x - 4)(x + 1) = 0$$

$$x - 4 = 0 \text{ and } x + 1 = 0$$

$$x - 4 = 0$$

$$x - 4 + 4 = 0 + 4$$

$$x = 4$$

$$x + 1 = 0$$

$$x + 1 - 1 = 0 - 1$$

$$x = -1$$

Tip: When you have an equation in factor form, disregard any factor that is a number and contains no variables. For example, in $4(x - 5)(x + 5) = 0$, disregard the 4. It will have no effect on your two solutions.

Solving Quadratic Equations by Using the Zero Product Rule

If a quadratic equation is not equal to zero, rewrite it so that you can solve it using the zero product rule.

Example C

To solve $x^2 + 9x = 0$, first factor it:

$$x(x + 9) = 0$$

Now you can solve it.

Either $x = 0$ or $x + 9 = 0$.

Therefore, possible solutions are $x = 0$ and $x = -9$.

C. Graphs of Quadratic Equations are Parabolas

1. Introduction to Parabolas

The (x,y) solutions to quadratic equations can be plotted on a graph. These graphs are called **parabolas**. Typically you will be presented with parabolas given by equations in the form of $y = ax^2 + bx + c$.

Notice that the equation $y = x^2$ conforms to this formula—both b and c are zero.

$$y = (1)x^2 + (0)x + (0) \text{ is equivalent to } y = x^2$$

The value of a cannot equal zero, however.

2. Movement of the Parabola on the Graph - Opening Up and Down

If a is greater than zero, the parabola will open upward. If a is less than zero, the parabola will open downward.

The x -coordinate of the turning point, or vertex, of the parabola is given by:

$$x = \frac{-b}{2a}$$

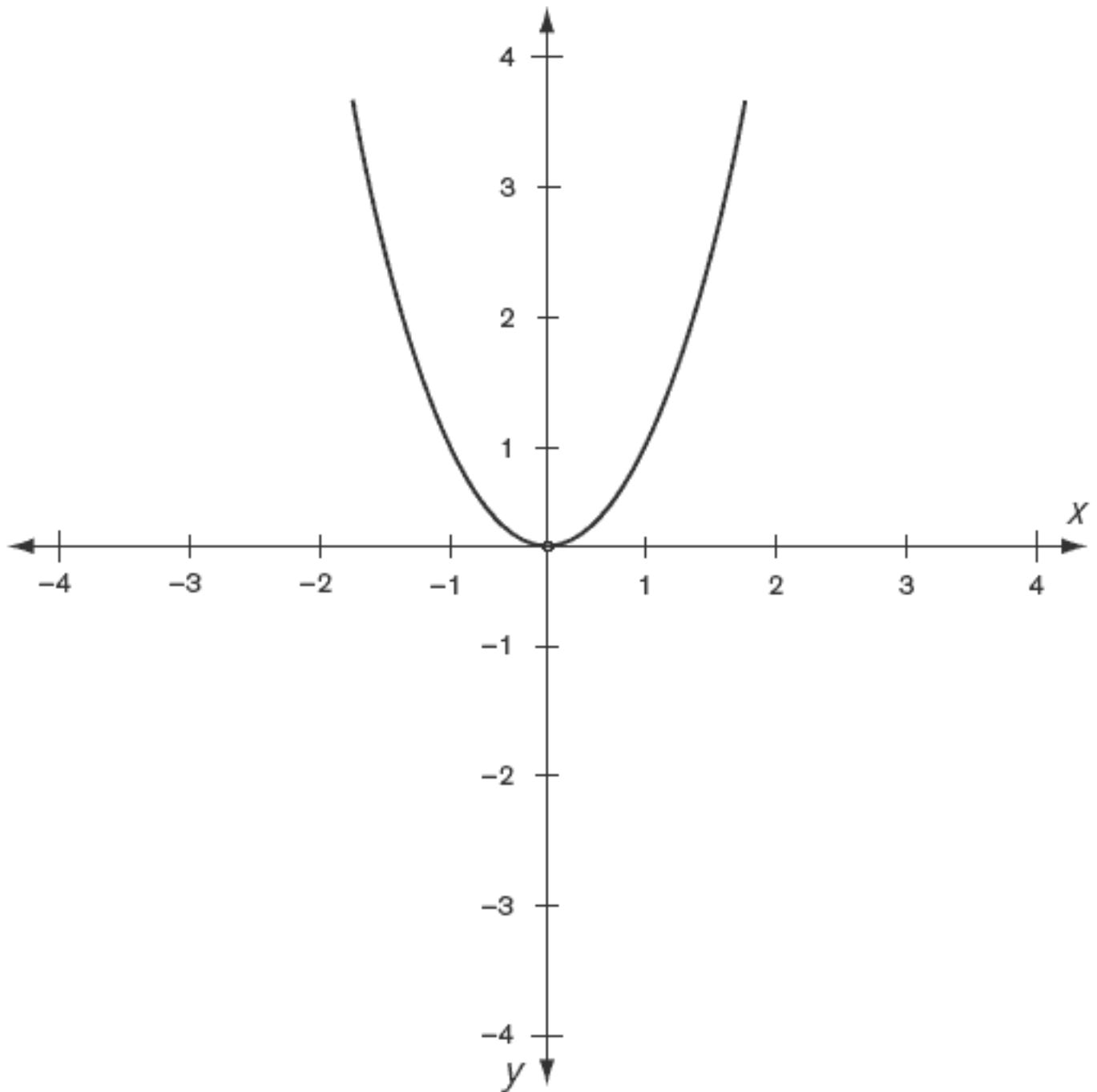
You can use this x -value in the original formula and solve for y (the y -coordinate of the turning point).

There will also be a line of symmetry given by:

$$x = \frac{-b}{2a}$$

For the graph $y = x^2$, $x = \frac{-b}{2a} = 0$. The line of symmetry is $x = 0$. The y -coordinate of the vertex is located at $y = x^2 = 0^2 = 0$, so the vertex is at $(0,0)$. Technically a parabola could also be given by the formula $x = ay^2 + by + c$.

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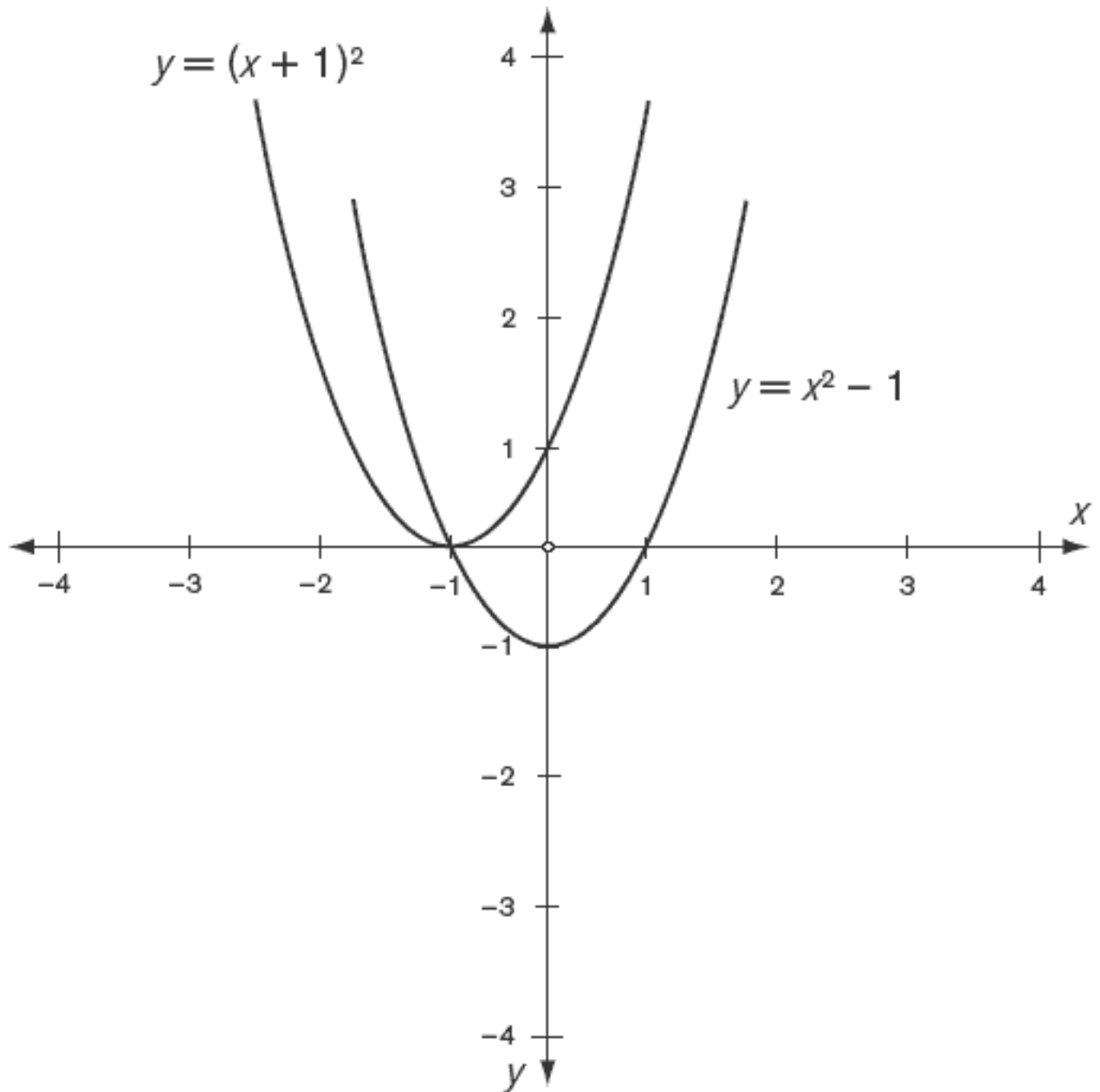
The graph of the equation $y = x^2$ is a parabola.

Because the x -value is squared, the positive values of x yield the same y -values as the negative values of x . The graph of $y = x^2$ has its vertex at the point $(0,0)$. The vertex of a parabola is the turning point of the parabola. It is either the minimum or maximum y -value of the graph. The graph of $y = x^2$ has its minimum at $(0,0)$. There are no y -values less than 0 on the graph.

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3. Movement of the Parabola on the Graph - Moving Up, Down, Left, and Right

The graph of $y = x^2$ can be translated around the coordinate plane. While the parabola $y = x^2$ has its vertex at $(0,0)$, the parabola $y = x^2 - 1$ has its vertex at $(0,-1)$. After the x term is squared, the graph is shifted down one unit. A parabola of the form $y = x^2 - c$ has its vertex at $(0,-c)$ and a parabola of the form $y = x^2 + c$ has its vertex at $(0,c)$.



The parabola $y = (x + 1)^2$ has its vertex at $(-1,0)$. The x -value is increased before it is squared. The minimum value of the parabola is when $y = 0$ (because $y = (x + 1)^2$ can never have a negative value).
 D. Legault, Minnesota Literacy Council, 2014

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value). The expression $(x + 1)^2$ is equal to 0 when $x = -1$. A parabola of the form $y = (x - c)^2$ has its vertex at $(c,0)$ and a parabola of the form $y = (x + c)^2$ has its vertex at $(-c,0)$.

What are the coordinates of the vertex of the parabola formed by the equation $y = (x - 2)^2 + 3$?

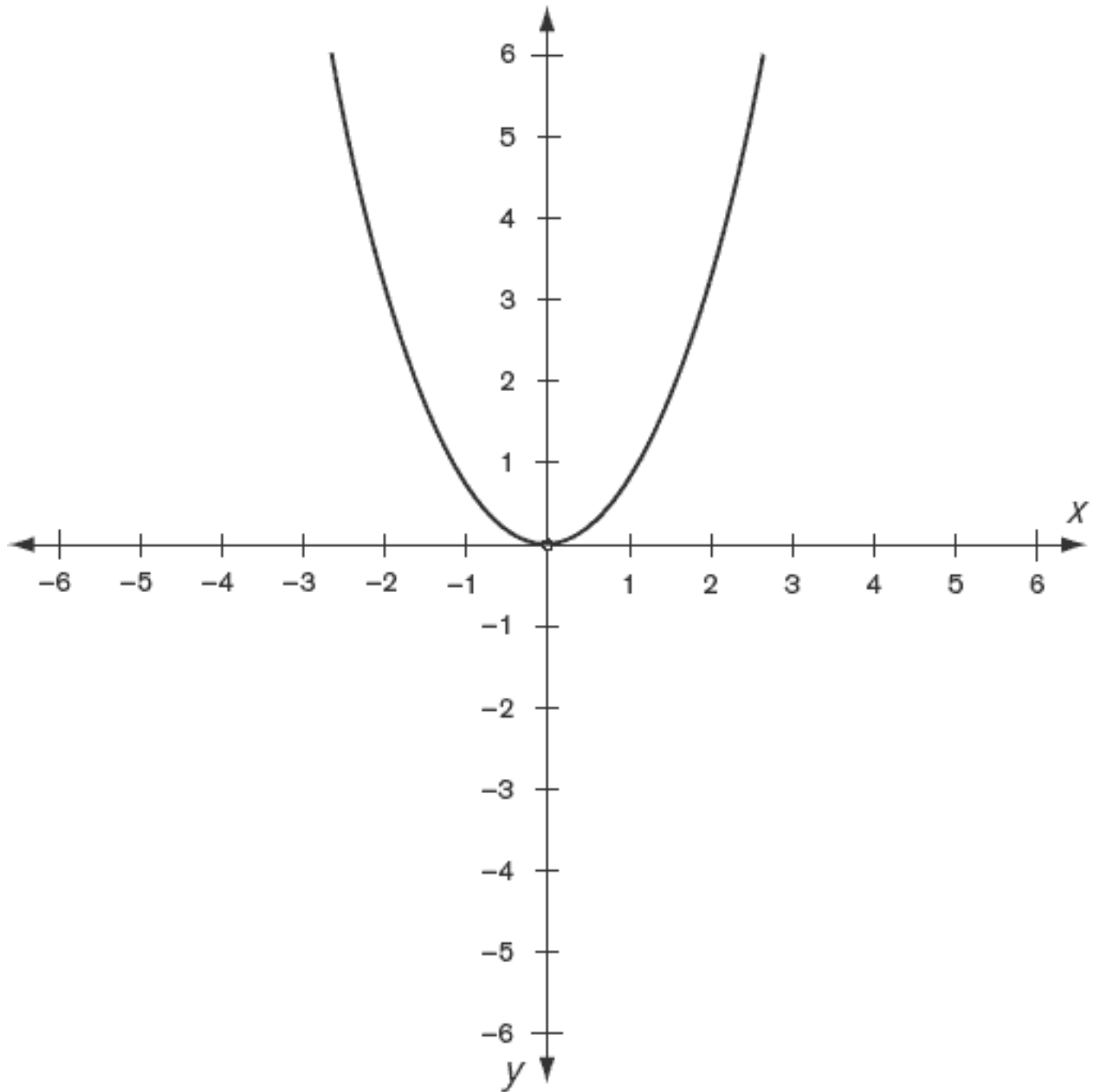
To find the x-value of the vertex, set $(x - 2)$ equal to 0: $x - 2 = 0$, $x = 2$. The y-value of the vertex of the parabola is equal to the constant that is added to or subtracted from the x squared term. The y-value of the vertex is 3, making the coordinates of the vertex of the parabola $(2,3)$.

If parabolas with the formula $y = x^2 + bx + c$ open upward or downward, how do you think parabolas given by the formula $x = ay^2 + by + c$ appear?

It is important to be able to look at an equation and understand what its graph will look like. You must be able to determine what calculation to perform on each x-value to produce its corresponding y-value.

For example, here is the graph of $y = x^2$.

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The equation $y = x^2$ tells you that for every x -value, you must square the x -value to find its corresponding y -value. Let's explore the graph with a few x -coordinates:

An x -value of 1 produces what y -value? Plug $x = 1$ into $y = x^2$. When $x = 1$, $y = 1^2$, so $y = 1$. So, you know a coordinate in the graph of $y = x^2$ is $(1,1)$.

An x -value of 2 produces what y -value? Plug $x = 2$ into $y = x^2$. When $x = 2$, $y = 2^2$, so $y = 4$. Therefore, you know a coordinate in the graph of $y = x^2$ is $(2,4)$.

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An x-value of 3 produces what y-value? Plug $x = 3$ into $y = x^2$. When $x = 3$, $y = 3^2$, so $y = 9$. That determines that a coordinate in the graph of $y = x^2$ is $(3,9)$.

Tip: Solving the formula of a parabola for x tells you the x intercept (or intercepts) of the parabola—that is, where the parabola crosses the x -axis. If you get two real values for x , the parabola crosses the x -axis at two points. If you get one real root, then that value is the vertex. If both roots are complex, then the parabola never crosses the x -axis.

4. Comparing Graphs of Parabolas

Compare the graph of $y = x^2$ with the graph of $y = (x - 1)^2$.

Let's compare what happens when you plug x -values into both equations.

$$y = x^2$$

$$\text{If } x = 1, y = 1.$$

$$\text{If } x = 2, y = 4.$$

$$\text{If } x = 3, y = 9.$$

$$\text{If } x = 4, y = 16.$$

$$y = (x - 1)^2$$

$$\text{If } x = 1, y = 0.$$

$$\text{If } x = 2, y = 1.$$

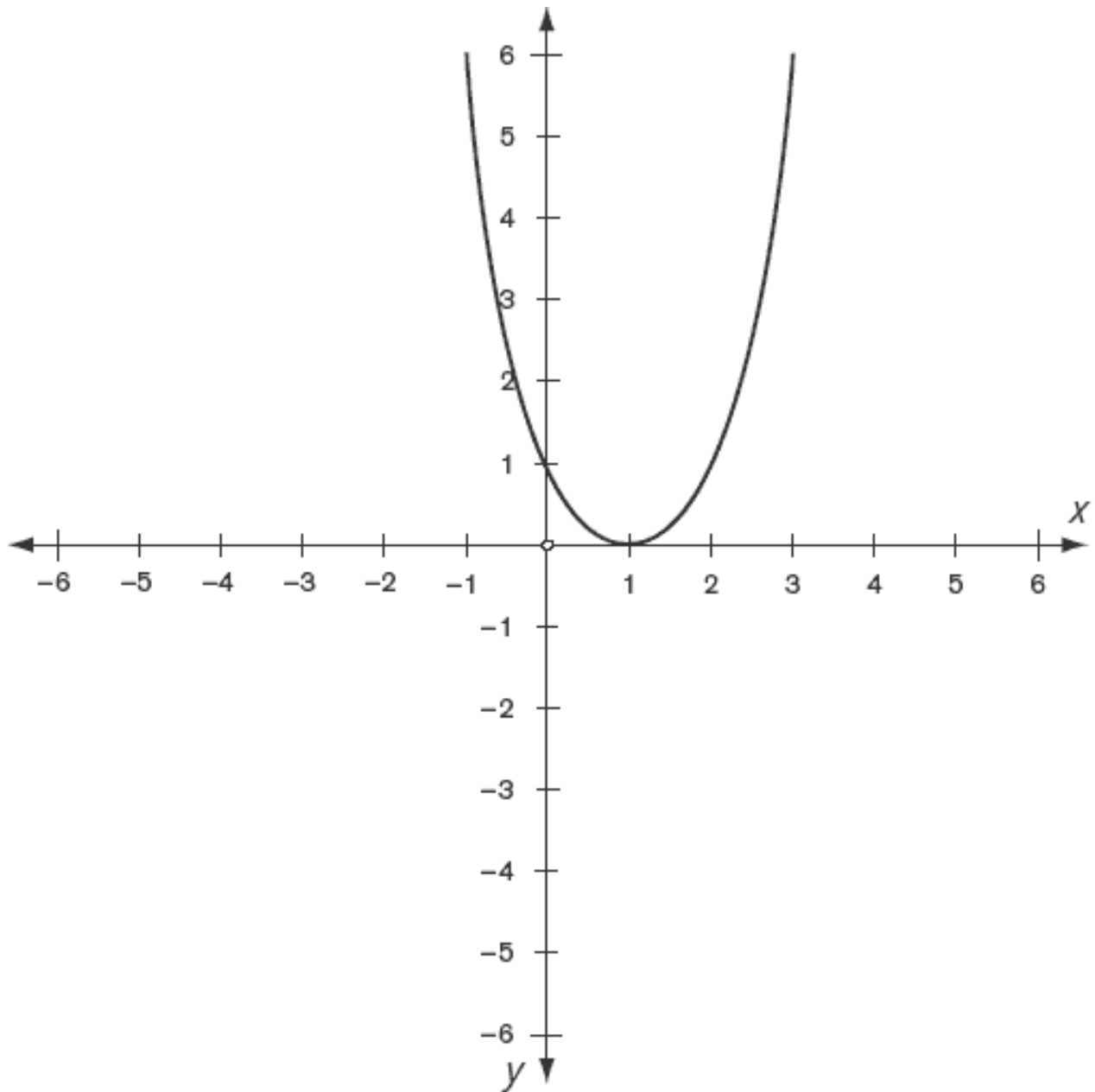
$$\text{If } x = 3, y = 4.$$

$$\text{If } x = 4, y = 9.$$

The two equations have the same y -values, but they match up with different x -values because $y = (x - 1)^2$ subtracts 1 before squaring the x -value.

The graph of $y = (x - 1)^2$ looks identical to the graph of $y = x^2$ except that the base is shifted to the right (on the x -axis) by 1:

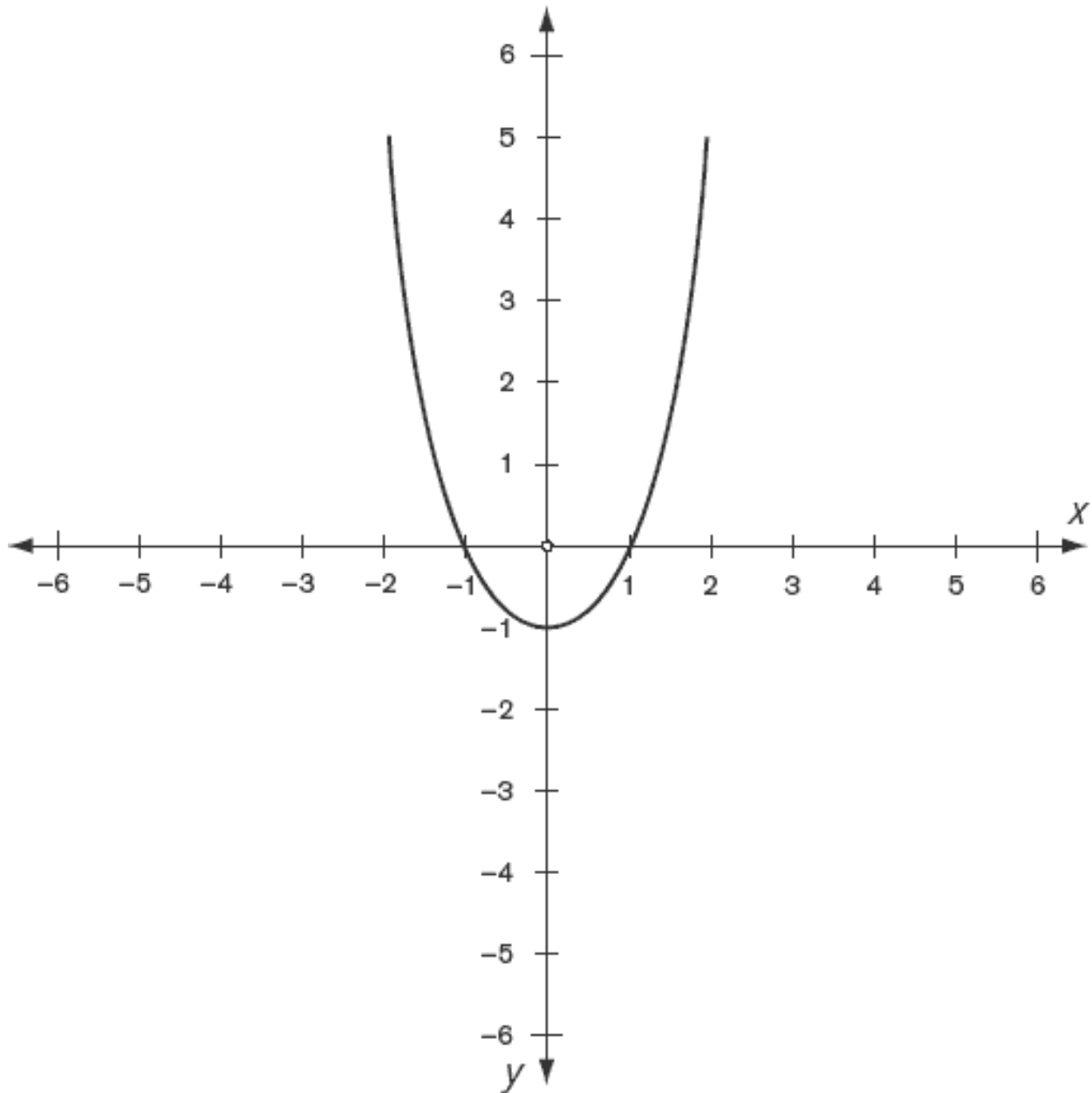
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How would the graph of $y = x^2$ compare with the graph of $y = x^2 - 1$?

The graph of $y = x^2 - 1$ looks identical to the graph of $y = x^2$ except that the base is shifted down (on the y-axis) by 1:

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5. Word Problems Aren't a Problem

You can easily solve the word problems using quadratic equations. Let's look carefully at an example.

Example

You have a patio that is 8 ft. by 10 ft. You want to increase the size of the patio to 168 square ft. by adding the same length to both sides of the patio. Let x = the length you will add to each side of the patio. You find the area of a rectangle by multiplying the length times the width. The new area of the patio will be 168 square ft.

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$$(x + 8)(x + 10) = 168$$

FOIL the factors $(x + 8)(x + 10)$.

Simplify.

Subtract 168 from both sides of the equation.

Simplify both sides of the equation.

Factor.

Set each factor equal to zero.

Solve the equation.

Subtract 22 from both sides of the equation.

Simplify both sides of the equation.

Solve the equation.

Add 4 to both sides of the equation.

Simplify both sides of the equation.

$$x^2 + 10x + 8x + 80 = 168$$

$$x^2 + 18x + 80 = 168$$

$$x^2 + 18x + 80 - 168 = 168 - 168$$

$$x^2 + 18x - 88 = 0$$

$$(x + 22)(x - 4) = 0$$

$$x + 22 = 0 \text{ and } x - 4 = 0$$

$$x + 22 = 0$$

$$x + 22 - 22 = 0 - 22$$

$$x = -22$$

$$x - 4 = 0$$

$$x - 4 + 4 = 0 + 4$$

$$x = 4$$

Because this is a quadratic equation, you can expect two answers. The answers are 4 and -22 . However, -22 is not a reasonable answer. You cannot have a negative length. Therefore, the only answer is 4.

To check your calculations, review the original dimensions of the patio—8 ft. by 10 ft. If you were to add 4 to each side, the new dimensions would be 12 ft. by 14 ft. When you multiply 12 times 14, you get 168 square ft., which is the new area you wanted.

6. Review of the Quadratic Formula

You can use the quadratic formula to solve quadratic equations. You might be asking yourself, "Why do I need to learn another method for solving quadratic equations when I already know how to solve them by using factoring?" Well, not all quadratic equations can be solved using factoring. You use the factoring method because it is faster and easier, but it will not always work. However, the quadratic formula will always work.

The quadratic formula is a formula that allows you to solve any quadratic equation—no matter how simple or difficult. If the equation is written in the form $ax^2 + bx + c = 0$, then the two solutions for x will

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

be $x =$. It is the \pm in the formula that gives us the two answers: one with $+$ in that spot, and one with $-$. The formula contains a radical, which is one of the reasons you studied radicals

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in a previous lesson. To use the formula, you substitute the values of a , b , and c into the formula and then carry out the calculations.

Example

$$3x^2 - x - 2 = 0$$

Determine a , b , and c . $a = 3$, $b = -1$, and $c = -2$

Take the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute in the values of a , b , and c .

$$\frac{-[-1] \pm \sqrt{-1^2 - 4(3)(-2)}}{2 \cdot 3}$$

Simplify.

$$\frac{1 \pm \sqrt{1 - [-24]}}{6}$$

Simplify more.

$$\frac{1 \pm \sqrt{25}}{6}$$

Take the square root of 25.

$$\frac{1 \pm 5}{6}$$

The solutions are 1

and $-\frac{2}{3}$

$$\frac{1+5}{6} = \frac{6}{6} = 1 \text{ and } \frac{1-5}{6} = -\frac{4}{6} = -\frac{2}{3}$$

Tip: To use the quadratic formula, you need to know the a , b , and c of the equation. However, before you can determine what a , b , and c are, **the equation must be in $ax^2 + bx + c = 0$ form**. For example, the equation $5x^2 + 2x = 9$ must be transformed to $ax^2 + bx + c = 0$ form ($5x^2 + 2x - 9 = 0$).

Worksheet 37.1

Kuta Software - Infinite Algebra 1

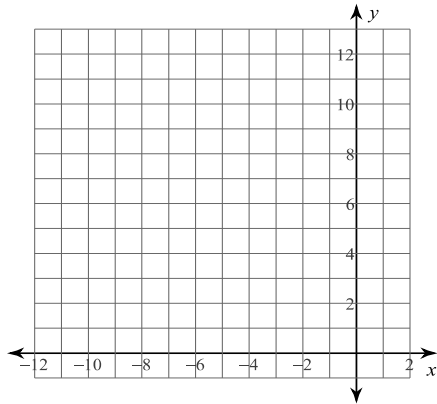
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Graphing Quadratic Functions

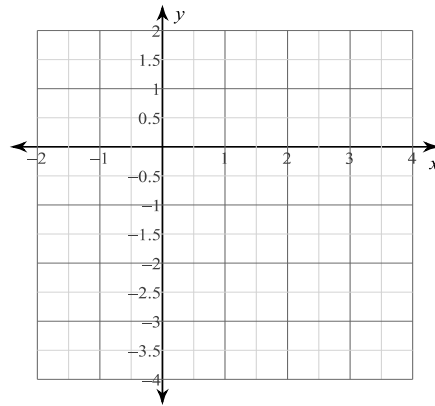
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Sketch the graph of each function.

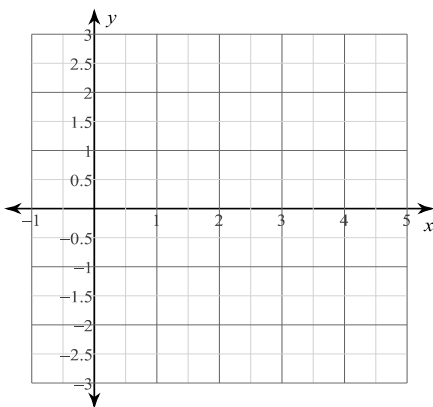
1) $y = 3x^2$



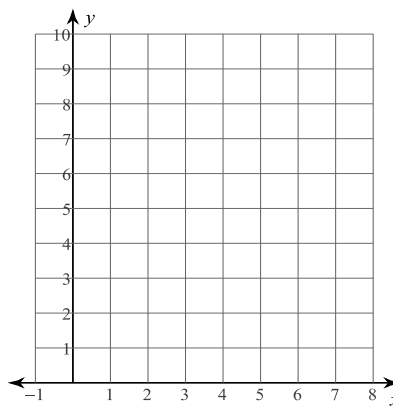
2) $y = -\frac{1}{2}x^2$



3) $y = -x^2 + 2x + 1$

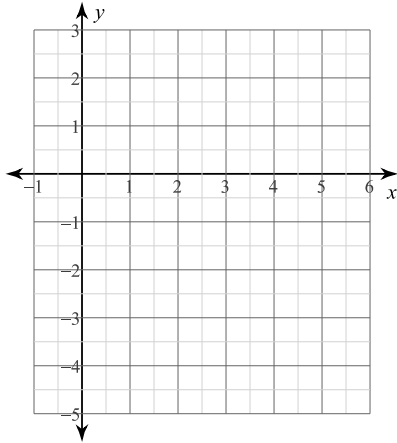


4) $y = 2x^2 - 16x + 33$

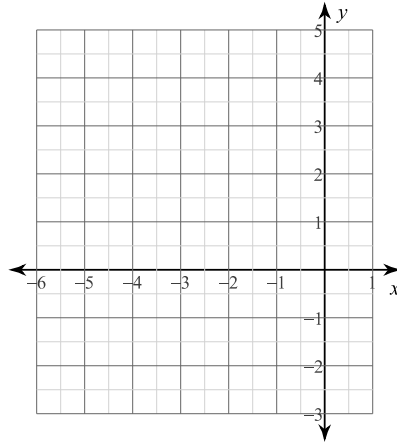


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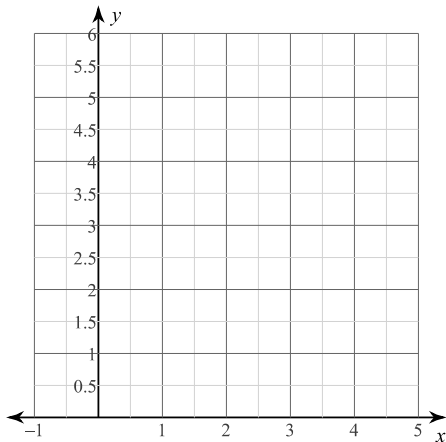
5) $y = x^2 - 8x + 13$



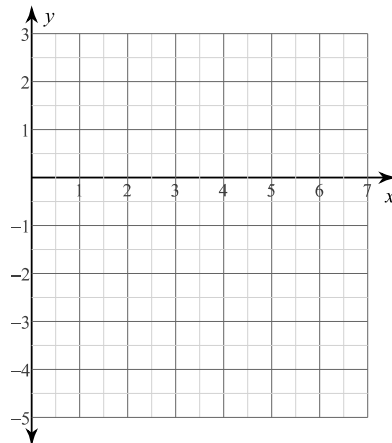
6) $y = -x^2 - 8x - 13$



7) $y = (x - 3)^2 + 1$



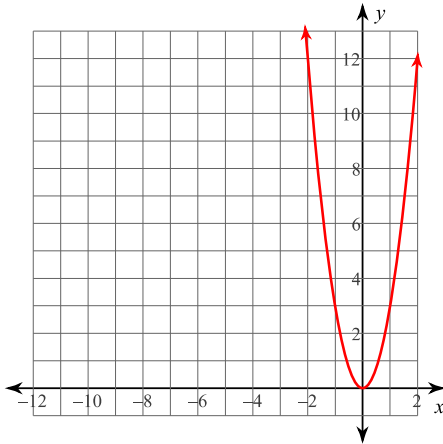
8) $y = \frac{1}{2}(x - 4)^2 - 2$



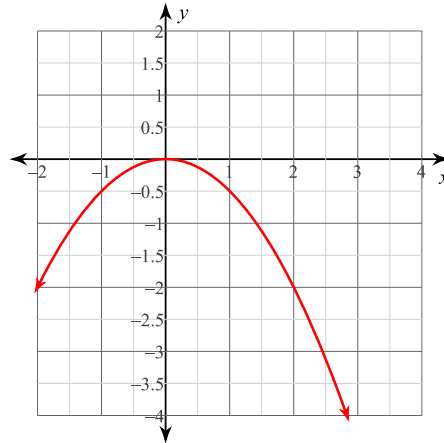
Worksheet 37.1 Answers

Sketch the graph of each function.

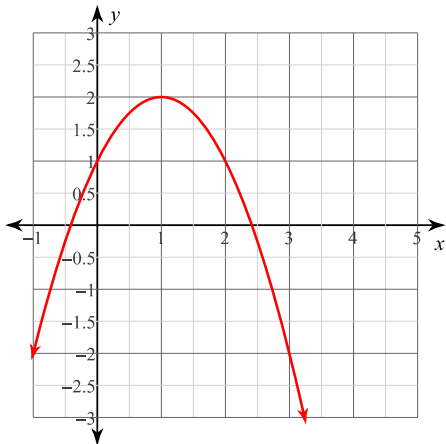
1) $y = 3x^2$



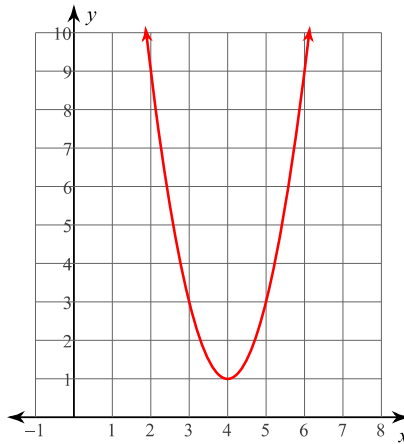
2) $y = -\frac{1}{2}x^2$



3) $y = -x^2 + 2x + 1$

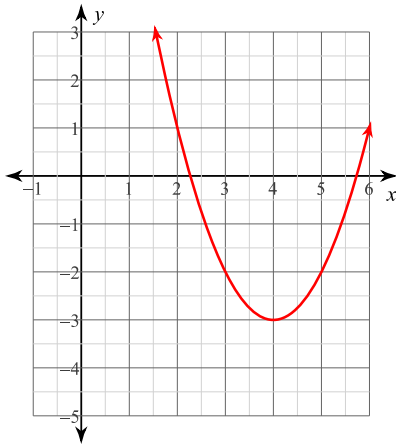


4) $y = 2x^2 - 16x + 33$

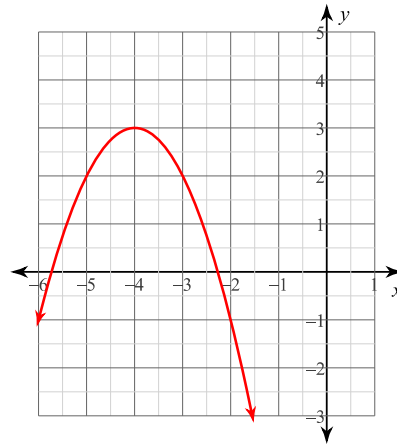


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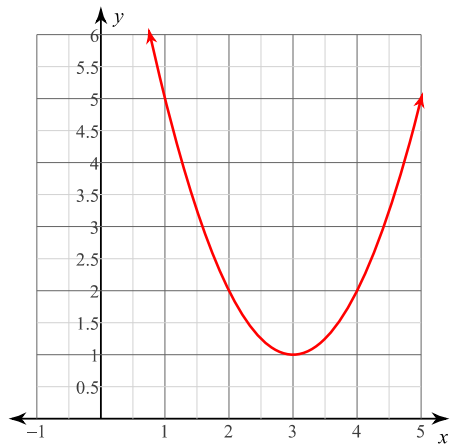
5) $y = x^2 - 8x + 13$



6) $y = -x^2 - 8x - 13$



7) $y = (x - 3)^2 + 1$



8) $y = \frac{1}{2}(x - 4)^2 - 2$

