Weekly Focus: exponents and scientific notation
Weekly Skill: calculate, word problems

LESSON 21: Exponents and Scientific Notation

Lesson Summary: For the Warm Up, students will solve a problem about brain cells. In Activity 1, they will add and subtract exponents. In Activity 2, they will multiply and divide exponents. In Activity 3, they will practice scientific notation. In Activities 4 and 5, they will do word problems. There are extra activity ideas provided at the end. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 21:

- Video (length 4:00) on scientific notation. The video is required for teachers and optional for students.
- 1 Notes handout (21.1)
- 4 Worksheets (21.2, 21.3, 21.4, 21.5) with answers (attached or embedded link)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 56 – 57)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 70 – 73)

Objectives: Students will be able to:

- Solve the brain word problem
- Do computation with exponents and scientific notation
- Solve problems with exponents and scientific notation

ACES Skills Addressed: N, CT, LS, ALS
CCRS Mathematical Practices Addressed: Building Solution Pathways, Mathematical Fluency, Attend to Precision
Levels of Knowing Math Addressed: Intuitive, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 21 Warm-up: Solve the brain questions	Time: 10 Minutes

Write on the board: An adult human brain weighs about 1400 grams. There are about 86 billion neurons (brain cells) in our brains.

Questions:

- Estimate how many pounds the brain weighs.
  - Hint: First convert grams to kg. (1000 g = 1 kg) so about 1 ½ kg.
  - Hint: 1 kg is a little more than 2 lbs. (2.2) so *1 ½ kg equals about 3 lbs*.
- Write out 86 billion as a number.
  - *86,000,000,000*
- For an adult who weighs 120 lbs., what percent of his weight does his brain weigh?
  - *3/120 = 0.025 = 2.5 %*

Lesson 21 Activity 1: Adding and Subtracting Exponents	Time: 10 Minutes

1. In working with exponents, we need to learn how to add, subtract, multiply, and divide them.

2. For adding and subtracting, we **combine like terms.** Like terms have the same variables raised to the same exponent.

3. Examples: $5x^2$ and x^2 are like terms. $5x^2$ and x^3 are not.

4. Example: $8y^3 + 2y^3 + 7 + 4x^2 - 2x^2 = 10y^3 + 2x^2 + 7$. Add more examples if needed.

5. Do **Worksheet 21.2** to practice.

Lesson 21 Activity 2: Multiplying and Dividing Exponents	Time: 25 Minutes

1) There is **Handout 21.1** with examples of the following. You many choose to explain the three examples below and have students practice those only and give them the handout notes later because it has more information than they need immediately.

2) Example of Multiplying Exponents: $3^2 \bullet 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 9 \times 81 = 729$. As you can see, this is the same as 3^6. When we multiply exponents, we can add the exponents before calculating. (Number 3 on handout notes).

3) Example of Dividing Exponents: $3^4/3^2 = 3 \times 3 \times 3 \times 3 / 3 \times 3 = 81/9 = 9$ **or** you can cancel before you multiply it out to get $3 \times 3 \times \cancel{3} \times \cancel{3} / \cancel{3} \times \cancel{3} = 9$. As you can see, this is the same as subtracting the second exponent from the first. We subtract exponents $4 - 2 = 2$, so answer is $3^2 = 9$. (Number 4 on handout notes).

4) Example of Negative Exponent: When the exponent is negative, it means it is the denominator in a fraction. Example: 4^{-3} means the same as $1/4^3$. (Number 2 on handout notes).

5) Practice with **Worksheets 21.3 and 21.4.** Do a few examples together first.

Lesson 21 Activity 3: Scientific Notation	Time: 20-25 Minutes

1. In changing from **standard notation** to **scientific notation**, we need to remember two rules:
   a. Scientific notation is written as a number between 1 and 10 multiplied by $10^{integer}$.
   b. We count the number of decimal places from where we want to have the decimal point to where it is. If we count to the right, the integer is positive. If we count to the left, the integer is negative.

2. Example A: $138{,}000 = 1.38 \times 10^5$. I count from where I want the decimal point to be (between the 1 and the 3 and count to the right 5 times to get the integer 5.

3. Example B: $0.000138 = 1.38 \times 10^{-4}$. This time I had to count 4 spaces to the left, so my integer is -4.

4. Example C: The same principles apply for converting from scientific to standard notation. Our brains contain 8.6×10^{10} neurons. The integer is positive (and we know we want a larger number) so we move the decimal point to the right 10 times to get 86,000,000,000.

5. Practice with **Worksheet 21.5.** Make sure to do some in each section.

Lesson 21 Activity 4: Practice Problems	**Time: 15-20 Minutes**
Do the problems in the **student book pages 56-57** together.	

Lesson 21 Activity 5: Word Problems	**Time: 35-45 Minutes**
Start the problems in the **workbook pages 70-73** Assign the rest as homework.	

Lesson 21 Extra Activity for Application	**Time: 45-50 Minutes**
There is an activity available on yummymath.com that includes exponents. Search for space shuttle (on the right hand side towards the middle of the page) and it will provide the activity as a pdf.	

Lesson 21 Extra: Card Game	**Time: 15+ Minutes**
Here is a card game that can be played in small groups to reinforce the meaning of exponents.	

Exponents Game

Exponents can be tricky at first, but this game provides plenty of practice! Draw from a deck of cards to find your numbers. Create a problem using exponents, and then use a calculator or scratch paper to work out the answers. Help each other out if someone gets stuck. The goal is to create problems that result in the largest products. You'll work on multiplication and improve your understanding of exponents at the same time!

What's an Exponent?

exponent: indicates how many times the number is to be used in multiplication; it is written as a small number at the upper right of a base number (e.g. 3^4 = 3 x 3 x 3 x 3, which comes out to 81.)

What You Need:

- Deck of cards (with the face cards removed)
- Several players
- Calculator
- Paper
- Pencil

What You Do:

1. Decide which player will be the dealer and which will be the scorekeeper.
2. The dealer should deal two cards to each player.
3. Explain that the goal of each round is to create an exponent with the highest value.
4. Each player should arrange the cards to indicate which number is the base and which is the exponent.
5. Encourage them to use the two numbers they've drawn to create a problem with exponents. For example, if a player drew a 3 and a 4, they could create the problem 3^4 or 4^3.
6. Tell the players that the goal is to create the exponent with the highest value.
7. Have the players use scratch paper to compute their answers and then use the calculator to double check their correctness. Using scratch paper puts the focus on the computation, and double checking their answers will help them learn how to use a calculator effectively.
8. The player with the highest value wins the round. Have the scorekeeper give the winning player a point.
9. Play as many rounds as you'd like. At the end of play add the totals up to see who's won!

Handout 21.1 Exponent Properties

1. Zero Exponent:

Any number raised to the zero power is equal to 1.

$a^0 = 1 \; ; \; a \neq 0$

Example: $4^0 = 1$ and $2500^0 = 1$

2. Negative Exponent:

Negative exponents indicate reciprocation, with the exponent of the reciprocal becoming positive.

$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n \; ; \; a \neq 0$

Example: $3^{-2} = \frac{1}{3^2}$ or $\frac{1}{4^{-3}} = 4^3$

3. Product of like bases:

To multiply powers with the same base, add the exponents and keep the common base.

$a^m a^n = a^{m+n} \; ; \; a \neq 0$

Example: $2^3 2^2 = 2^5 = 32$

4. Quotient of like bases:

To divide powers with the same base, subtract the exponents and keep the common base.

$\frac{a^m}{a^n} = a^{m-n} \; ; \; a \neq 0$

Example: $\frac{3^5}{3^3} = 3^{5-3} = 3^2$

5. Power to a power:

To raise a power to a power, keep the base and multiply the exponents.

$$(a^m)^n = a^{mn}$$

Example: $(2^2)^3 = 2^{2x3} = 2^6$

6. Product to a power:

To raise a product to a power, raise each factor to the power.

$$(ab)^m = a^m b^m \; ; \; a \neq 0$$

Example: $(2x3)^2 = 2^2 \text{ x } 3^2$

7. Quotient to a power:

To raise a quotient to a power, raise the numerator and the denominator to the power.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \; ; \; a \neq 0$$

Example: $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$

8. Rational Exponent:

The denominator of the rational exponent becomes the index of the radical, and the numerator becomes the exponent of the radicand.

$$(a)^{\frac{x}{y}} = \sqrt[y]{a^x}$$

Example: $(2)^{\frac{2}{3}} = \sqrt[3]{2^2}$

Worksheet 21.3 Multiplying Exponents

Simplify. Your answer should contain only positive exponents.

1) $9k^{5} \cdot 3k^{3}$

2) $5yw^{-2} \cdot 7y^{-4}w^{5}$

3) $5n^{5}g^{-4} \cdot 6n^{-5}g^{6}$

4) $2y^{2}b^{6} \cdot 9y^{4}b^{3}$

5) $wc \cdot 4w^{3}c^{6}$

8) $dw \cdot 5d^{-4}w^{-6}$

9) $3s \cdot 4s^{-6}$

10) $5b^{3} \cdot 6b^{-5}w^{5}$

11) $\left(\frac{1}{3}\right)^{5} \cdot \left(\frac{1}{3}\right)^{2}$

12) $5z^{6} \cdot 6z^{-6} \cdot 9z^{-4}$

Worksheet 21.3 Answers

1) $9k^5 \cdot 3k^3$

$27k^8$

2) $5yw^{-2} \cdot 7y^{-4}w^5$

$35\frac{w^3}{y^3}$

3) $5n^5g^{-4} \cdot 6n^{-5}g^6$

$30\,g^2$

4) $2y^2b^6 \cdot 9y^4b^3$

$18y^6b^9$

5) $wc \cdot 4w^3c^6$

$4w^4c^7$

8) $dw \cdot 5d^{-4}w^{-6}$

$\frac{5}{d^3w^5}$

9) $3s \cdot 4s^{-6}$

$\frac{12}{s^5}$

10) $5b^3 \cdot 6b^{-5}w^5$

$\frac{30w^5}{b^2}$

11) $\left(\frac{1}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^2$

$\left(\frac{1}{3}\right)^7$

12) $5z^6 \cdot 6z^{-6} \cdot 9z^{-4}$

$\frac{270}{z^4}$

Worksheet 21.4 Divide Exponents

Simplify. Your answer should contain only positive exponents.

1) $\frac{6n^{-3}}{2n^{-6}}$

2) $\frac{4y}{3y^{-6}}$

3) $\frac{4^3}{4}$

4) $\frac{4dg^5}{2d^2g^3}$

5) $\frac{4b^4}{6b}$

6) $\frac{4y^{-6}}{7y^3r^{-3}}$

7) $\frac{5^5}{5^3}$

8) $\frac{c^6}{c^{-4}}$

9) $\frac{4b}{2b^5}$

10) $\frac{3^5}{3^{-2}}$

11) $\frac{6b^{-3}}{2b}$

12) $\frac{h}{h^4}$

Worksheet 21.4 Answers

1) $\frac{6n^{-3}}{2n^{-6}}$

$3n^{3}$

2) $\frac{4y}{3y^{-6}}$

$\frac{4y^{7}}{3}$

3) $\frac{4^{3}}{4}$

4^{2}

4) $\frac{4dg^{5}}{2d^{2}g^{3}}$

$\frac{2g^{2}}{d}$

5) $\frac{4b^{4}}{6b}$

$\frac{2b^{3}}{3}$

6) $\frac{4y^{-6}}{7y^{3}r^{-3}}$

$\frac{4r^{3}}{7y^{9}}$

7) $\frac{5^{5}}{5^{3}}$

5^{2}

8) $\frac{c^{6}}{c^{-4}}$

c^{10}

9) $\frac{4b}{2b^{5}}$

$\frac{2}{b^{4}}$

10) $\frac{3^{5}}{3^{-2}}$

3^{7}

11) $\frac{6b^{-3}}{2b}$

$\frac{3}{b^{4}}$

12) $\frac{h}{h^{4}}$

$\frac{1}{h^{3}}$

Worksheet 21.5 Scientific Notation

Write each number in standard format.

1) $3.17 \times 10^{-1} =$ _____

2) $9.1276 \times 10^{-8} =$ _____

3) $3.4151 \times 10^{3} =$ _____

4) $9.96 \times 10^{-3} =$ _____

5) $2.2214 \times 10^{-2} =$ _____

6) $7.973 \times 10^{2} =$ _____

7) $8.287 \times 10^{-6} =$ _____

8) $8.0718 \times 10^{4} =$ _____

9) $6.0946 \times 10^{7} =$ _____

10) $3.049 \times 10^{5} =$ _____

Write each number in scientific notation.

11) 6908100000 = _____

12) 0.00000000853520 = _____

13) 253600000 = _____

14) 0.00000089790 = _____

15) 89800 = _____

16) 2020000 = _____

17) 0.000222 = _____

18) 0.0009159 = _____

19) 0.000082650 = _____

20) 54.6 = _____

Worksheet 21.5 Answers

Write each number in standard format.

1) $3.17 \times 10^{-1} = 0.317$

2) $9.1276 \times 10^{-8} = 0.0000000912760$

3) $3.4151 \times 10^{3} = 3415.1$

4) $9.96 \times 10^{-3} = 0.00996$

5) $2.2214 \times 10^{-2} = 0.022214$

6) $7.973 \times 10^{2} = 797.3$

7) $8.287 \times 10^{-6} = 0.0000082870$

8) $8.0718 \times 10^{4} = 80718$

9) $6.0946 \times 10^{7} = 60946000$

10) $3.049 \times 10^{5} = 304900$

Write each number in scientific notation.

11) $6908100000 = 6.9081 \times 10^{9}$

12) $0.00000000853520 = 8.5352 \times 10^{-9}$

13) $253600000 = 2.536 \times 10^{8}$

14) $0.00000089790 = 8.979 \times 10^{-7}$

15) $89800 = 8.98 \times 10^{4}$

16) $2020000 = 2.02 \times 10^{6}$

17) $0.000222 = 2.22 \times 10^{-4}$

18) $0.0009159 = 9.159 \times 10^{-4}$

19) $0.000082650 = 8.265 \times 10^{-5}$

20) $54.6 = 5.46 \times 10^{1}$