Weekly Focus: probability
Weekly Skill: counting and problem solving

LESSON 14: Probability

Lesson Summary: In the warm up activity, students will solve a word problem about mean and median. In Activity 1, they will do some vocabulary. In Activity 2, they will practice counting probability. In Activity 3, they will count possible outcomes. In Activity 4, they will experiment possible outcomes in a game of rock, paper, scissors. In Activity 5, they will look at examples of dependent vs. independent events and do problems in the student book. In Activity 6, students will do word problems in the workbook. For the application, they will solve some real life problems based on probable outcomes. Lastly, they will do a simple exit ticket. There is also an extra word problem at the end. Estimated time for the lesson is two hours.

Materials Needed for Lesson 14:

- Video (about 4 minutes) on theoretical vs. experiential probability. The video is required for teachers and optional for students.
- Worksheet 14.1 with answers on counting basic probability with candy (attached)
- Worksheet 14.2 for the partner activity using rock, paper, and scissors (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 32-33) and Workbook (pages 42-45)
- Worksheet 14.3—Explanation of Dependent and Independent Events handout for Activity 5 (attached)
- Handout 14.4—Example of tree diagram to show possible outcomes of events (attached at the end—optional to use)
- Exit ticket

Objectives: Students will be able to:

- Explain what probability is
- Do some basic probability counting problems
- Work with a partner to count experimental probability
- Solve various probability word problems
- Understand when probability is used in real life

ACES Skills Addressed: N, CT, LS
CCRS Mathematical Practices Addressed: Model with Math, Use Tools Strategically, Make Sense of Problems and Persevere in Solving Them
Levels of Knowing Math Addressed: Intuitive, Concrete, and Abstract

<u>**Notes:**</u>
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 14 Warm-up: Solve the pizza problem	Time: 10 Minutes

Write on the board: Maggie wants to order large pizzas for a party and calls 4 restaurants. The prices she is quoted are \$11, \$13, \$10, and \$9.

Basic Questions:

- What is the average (mean) price of the 4 pizzas? (11+13+10+9=43/4=\$10.75)
- What is the range of prices? (\$9 to \$13 = a range of \$4)

Extension: Maggie calls a 5th restaurant and now announces to her friends that the median price overall is \$11 but doesn't give the price at the 5th restaurant.

- What do you know for sure about the price at the 5th restaurant?
  - It is greater than or equal to \$11 because there are two prices less than \$11, and for \$11 to be in the middle, the other two prices must be greater than or equal to \$11.

Lesson 14 Activity 1: Vocabulary True or False	Time: 5 Minutes

1. Probability is the chance of something happening.
2. Probability can only be measured as a decimal.
3. Experimental probability is when you actually try something to see what the outcome will be.
4. Theoretical probability is used to predict if something is likely

to happen.

5. Probability can be more than 100%.

Answers: *1.T, 2.F (can be fraction or %), 3.T, 4.T, 5.F (it is always between 0 and 1 or 0% and 100%)*

Lesson 14 Activity 2: Counting Simple Probability	**Time: 15 Minutes**

1) Probability is always a number between 0 and 1.

   A probability of 0 means it is impossible
   A probability of 1 means it is certain

2) To calculate probabilities, we do $\frac{\text{What we want to happen}}{\text{Total number of outcomes}}$

3) Example A: What is the probability that a baby will be born a boy? Since there are only 2 possible outcomes (boy or girl) the chances are 1 in 2 or 50-50 or 50%. If there are 82 babies born one day, how many are likely to be boys? (1/2 of 82 is 41). This is called *theoretical* probability because we did not actually count the babies; we just know the chances.

4) Example B: What are the chances that tomorrow morning you will wake up speaking Chinese? (0 out of 1 = 0 probability) (unless you do speak Chinese!)

5) Example C: What is the probability that the sun will rise tomorrow? (It is 1/1 so 1 or 100%)

6) Example D: What is the probability of Minnesota getting snow in the winter? (Close to 100% if not 100%. I did not find data to confirm or deny. We would say "very likely".)

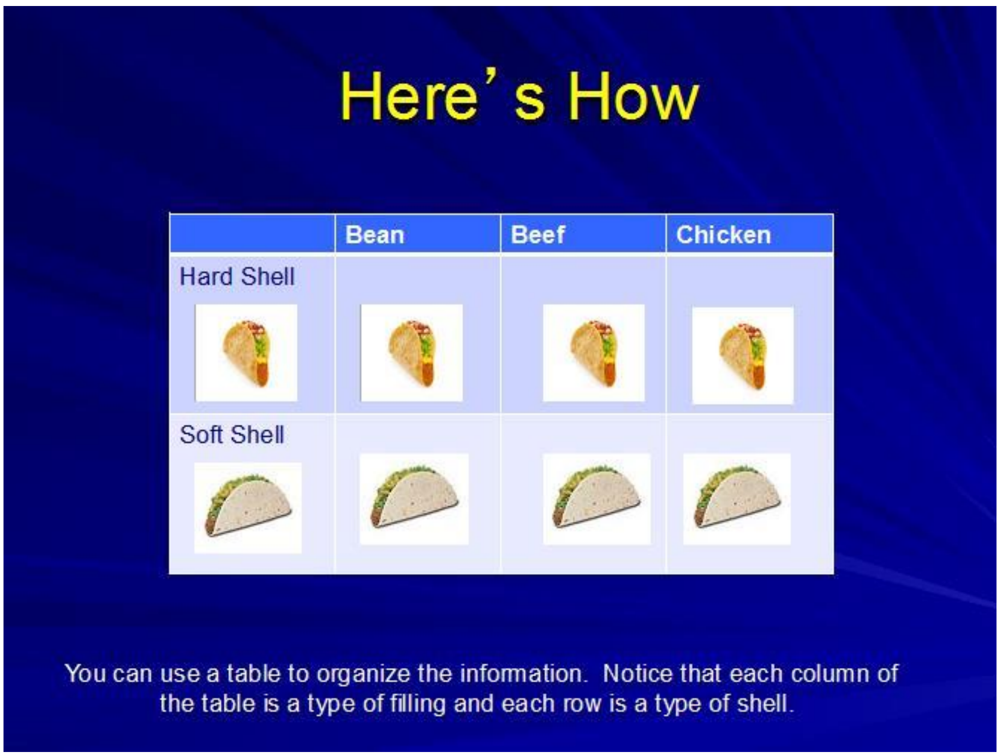
7) Do **Worksheet 14.1** to practice basic probability. Start together.

Lesson 14 Activity 3: Counting Possible Outcomes	Time: 10 Minutes

1) Before determining the probability, we must be able to count the total possible outcomes of an event. The previous examples were simple probability. Sometimes we have to count the total possible outcomes of two or more items.

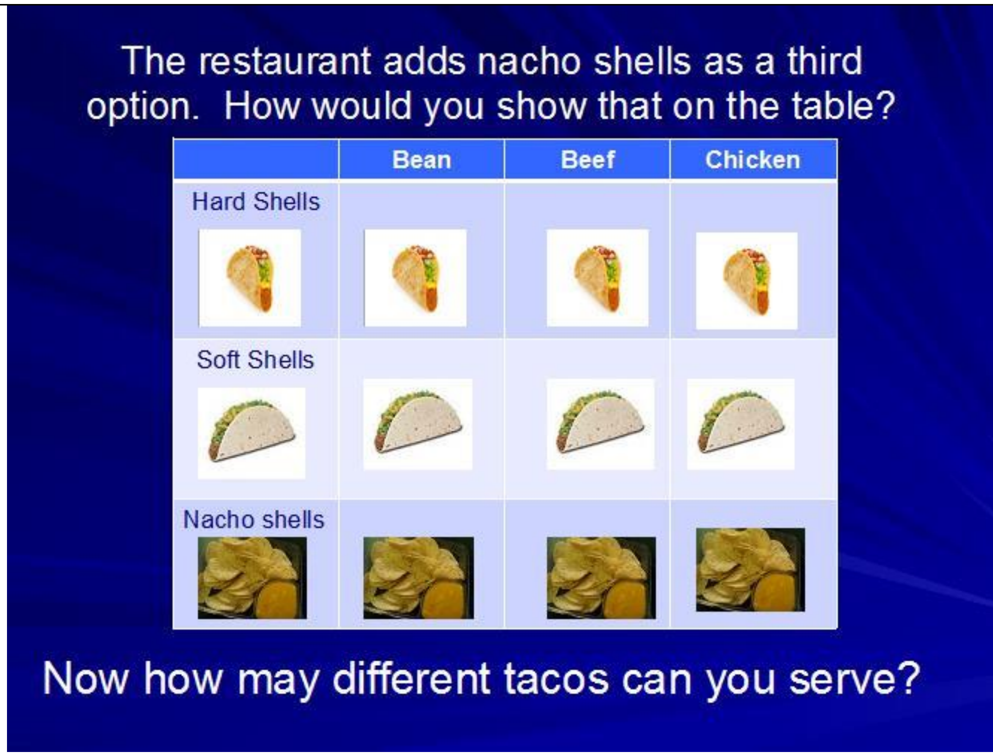
2) Example A: You work at a Mexican restaurant. Tacos are served in either hard shell or soft shell and filled with beans, chicken, or beef. How many combinations are there?

   Note to Teacher: Make a table like this on the board or show this one:



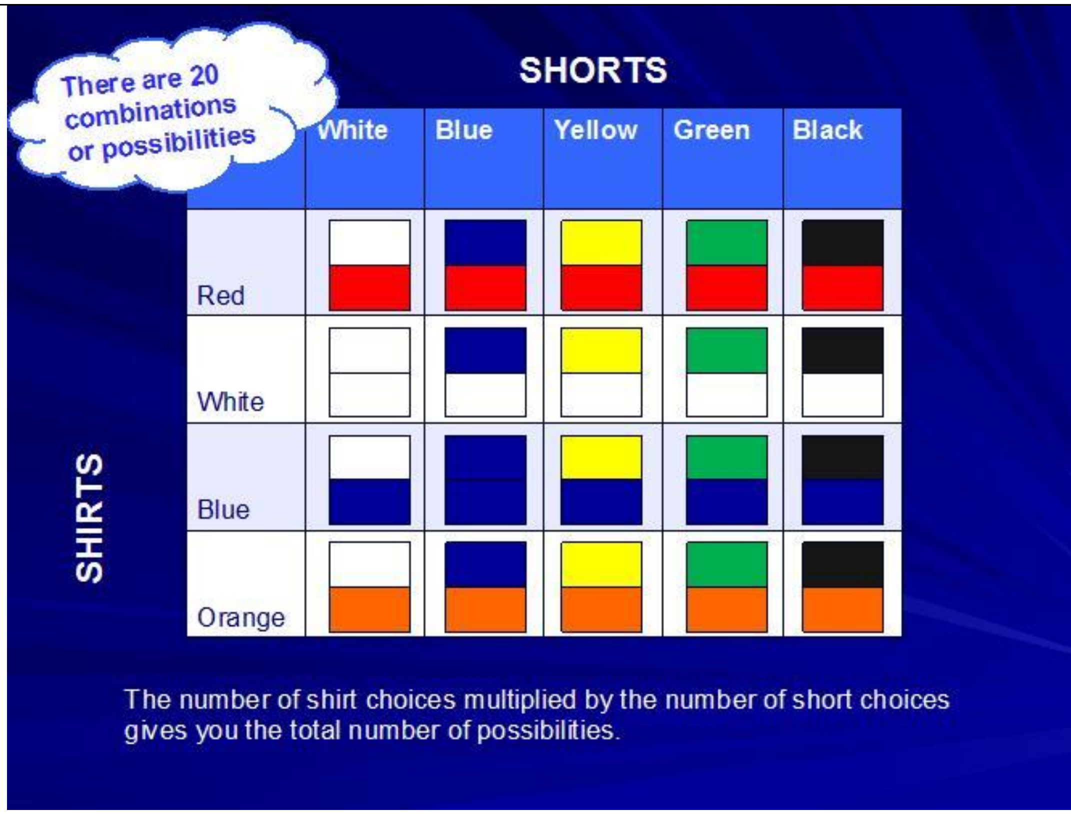
   As you can see, there are 6 combinations.

3) What would happen if you added nacho shells as a third option?



How many combinations can you serve? Do you see a pattern? *3 x 3 =9 options. The previous example was 2 x3 = 6.*

4) Example B: You are choosing new uniforms for your soccer team. There are 5 colors available for shorts and 4 colors for shirts. How many combinations are there? (4x5=20)



<u>Note to teacher:</u> Another way to depict the possibilities is by **tree diagrams** (see poster example and coin toss example attached).

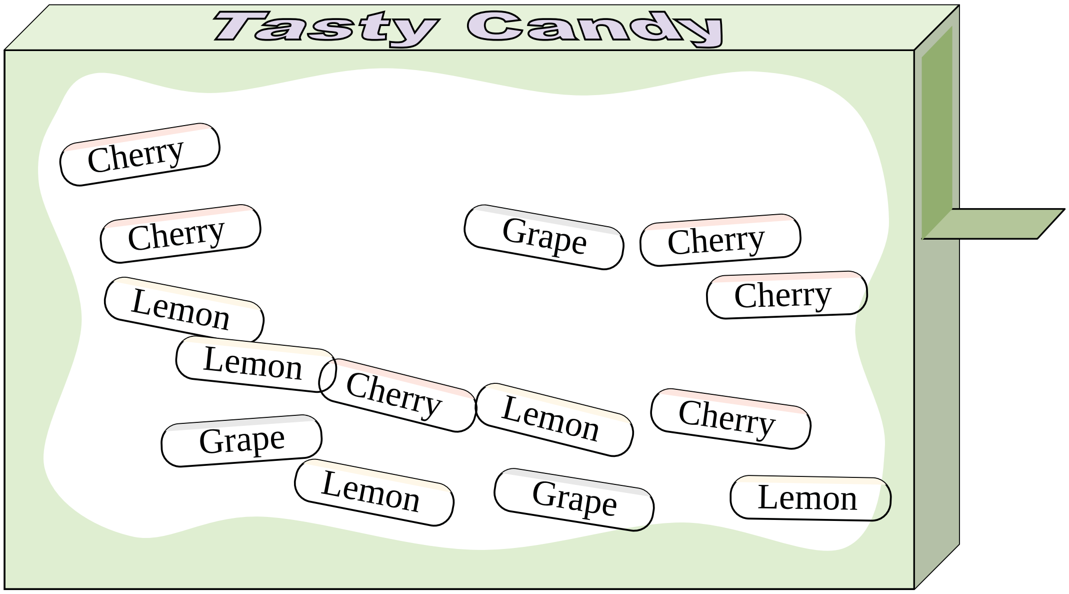
5) <u>Example C:</u> You are making sandwiches for a picnic. There are two types of bread, white and wheat, and four choice of filling: chicken, veggies, beef, or egg salad. How many different types of sandwiches can you make? *(2x4=8 possible outcomes)*

Lesson 14 Activity 4: Experimental Probability	**Time: 10-15 Minutes**

Use the attached **Worksheet 14.2 on Rock, Paper, or Scissors.** Have students work in pairs to play and count the **outcomes** (results) of their experimental probability. What did they notice?

Worksheet 14.1 Probability Practice with Candy

Use the candy box to solve each problem.



1) How many total pieces of candy are in the box?

2) What is the probability of selecting a cherry piece?

3) What is the probability of selecting a lemon piece?

4) What is the probability of selecting a grape piece?

5) If you picked 1 piece of candy out of the box which flavor would you have the highest probability of selecting?

6) Which flavor has the lowest probability of being selected?

7) If you picked a piece at random would you be more likely to select, a lemon piece of a cherry piece?

8) What is the probability of selecting either a cherry piece OR a grape piece?

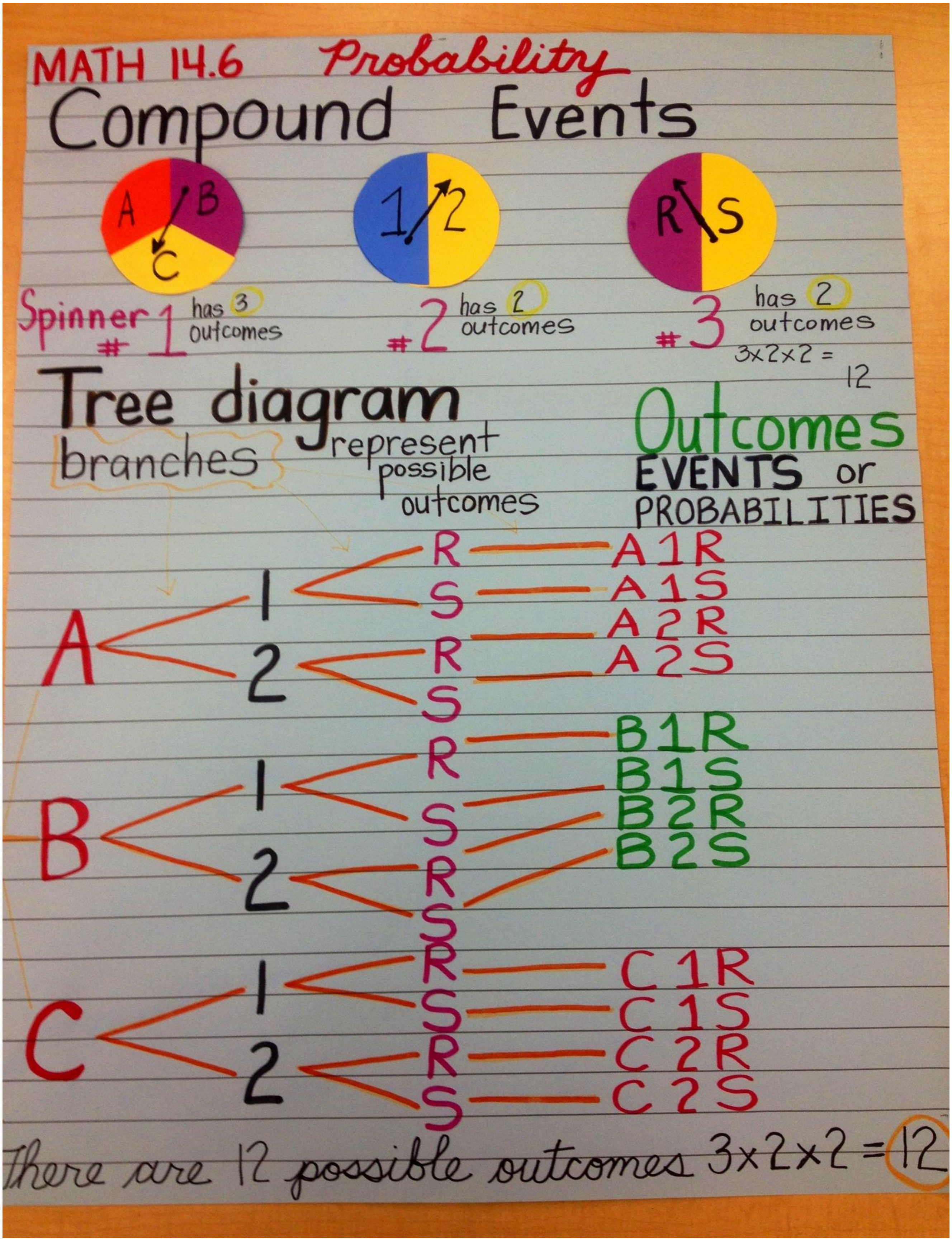
9) Your friend wants either a cherry piece or a grape piece. If you picked a piece out randomly, which one would you have the highest probability of selecting?

10) If you ate 4 lemon pieces, 4 cherry pieces and 2 grape pieces, which flavor would you have the highest probability of selecting next?

Worksheet 14.1 Probability Practice with Candy Answers

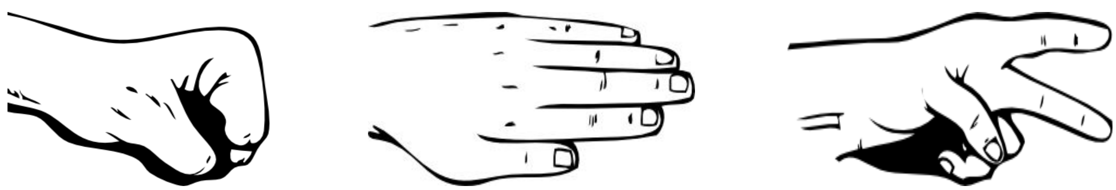
Answers

1. 14
2. 6 out of 14
3. 5 out of 14
4. 3 out of 14
5. cherry
6. grape
7. cherry
8. 9 out of 14
9. cherry
10. cherry


MATH 14.6 Probability
Compound Events
A B C
1 2
R S
Spinner #1 has 3 outcomes
#2 has 2 outcomes
#3 has 2 outcomes
3×2×2 = 12
Tree diagram
branches represent possible outcomes
Outcomes
EVENTS or PROBABILITIES
A 1 R
A 1 S
A 2 R
A 2 S
B 1 R
B 1 S
B 2 R
B 2 S
C 1 R
C 1 S
C 2 R
C 2 S
There are 12 possible outcomes 3×2×2=12

Worksheet 14.2 Probability Practice with Rock, Paper, Scissors

Probability Rock, Paper, Scissors



Probability: The chance that something will happen.

If you and a friend are playing rock, paper, scissors...

1. What is the probability that your friend will throw a rock?

2. What is the probability that your friend will not throw paper?

Get together with a partner and play rock, paper, scissors. Play a total of 20 times and record your data.

1. How many times was rock thrown by your partner? ______

   A. What was the probability? ______ / ______

2. How many times was paper thrown by your partner? ______

   A. What was the probability? ______ / ______

3. How many times was scissors thrown by your partner? ______

   A. What was the probability? ______ / ______

4. What do these results tell you? ______________________________

____

Lesson 14 Activity 5: Dependent and Independent Events	Time: 20 Minutes

1) Example A: I am eating lunch with a friend. I am having a sandwich and she is having soup. Does what I eat affect what she eats? *No, these are **independent** events.*

2) Example B: This time we are sharing a meal. I take part of the sandwich and give her the rest. Does what I eat affect what she eats? *Yes, it does because she gets what I didn't take. The second event is **dependent** on the first one.*

3) When there are two events happening, it is called compound probability. If the events are independent, you count the probability of Event A times Event B. If they are dependent, you count the probability of Event A times the probability of Event B after A is done. **Give the attached explanations of dependent and independent events to students.**

4) Do the problems on **pages 32-33 in the student book**. Do them together because there will be lots of explanation necessary.

5) Note to teacher: Be familiar with the answers and explanations to the questions (in the back of the book) before you teach class.

Lesson 14 Activity 6: Word Problems	Time: 20 Minutes

1) There are other types of questions that may or may not limit the number of choices.

2) Example A: Nicole is choosing a 4-digit PIN number for her bank account. Each digit can be a number from 0 to 9 and the numbers can repeat. What is the total number of possible PINs? *There are 10 possibilities each time so the total is 10x10x10x10= 10,000 possibilities.*

3) Example B: Ben is also choosing a 4-digit PIN number with digits 0-9, but the numbers cannot repeat. What is the total number of possible PINs? *There are 10 possibilities for the first digit but only 9 possibilities for the second digit (since one is used already) and so on. The answer is 10 x9x8x7= 5,040 possible combinations.*

4) Do the word problems in the **workbook pages 42-45.**

Lesson 14 Activity 7 Application: Thoughts on Probability	Time: 10 Minutes

Probability is used in many aspects of life: insurance, business, sports, gambling, dating websites, healthcare decisions, and others. Here are a few fun questions using probability:

1) A baseball player's batting average is calculated by the number of hits he or she gets when at-bat to get a percentage. If a player gets 3 hits for every 10 at-bats, his or her batting average is 3/10 or 30%. For baseball stats, we multiply all the percentages by 10, so a 30% probability translates to a 300 batting average.
   - *What would a 200 batting average mean? (The player's probability is 2/10 of getting a hit.)*

2) A researcher measured the probability of finding one's true love as 1 in 285,000. If the population of the world is 7,000,000,000, how many soul mates are there for you?
   - *Set up a proportion to solve:* $\frac{1}{285{,}000} = \frac{x}{7{,}000{,}000{,}000}$. *X=24,561 soul mates for you!*

3) Insurance companies use probability to decide how much to charge for coverage. Why does a young person's car insurance usually cost more than an older person?
   - *Because they are more likely to have accidents*

4) Why does the young person's health insurance cost less than the older person?
   - *Because the probability of the young person using health care is lower*

Lesson 14 Exit Ticket	Time: 5 Minutes

Say to the students: *"What is the probability that I will let you go home early today/tonight? "* Ask them how many times it happens in one month if they have math class with you 4 times a month. They should give a reasonable answer. ☺

Lesson 14 Extra Problem: Probability of Blood Type	Time: 3 Minutes

The blood groups of 200 people are distributed as follows: 50 have type **A** blood, 65 have **B** blood type, 70 have **O** blood type and 15 have type **AB** blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

Answer: 70/200= 0.35

Worksheet 14.3—Independent Events

Independent Events

Independent Events are not affected by previous Events.

A coin does not "know" it came up heads before ...

... each toss of a coin is a perfect isolated event.

When rolling a pair of dice, one die does not affect the outcome of the other die ...

... each die is an isolated event.

$$\textbf{Probability of an event happening} = \frac{\textbf{Number of ways it can happen}}{\textbf{Total number of outcomes}}$$

Probability of getting a "Head" when tossing a coin?

$$P(\text{Head}) = \frac{\text{"Head"}}{\text{"Head and Tail"}} = \frac{1}{2}$$

Probability of rolling a "4" on a die?

$$P(4) = \frac{\text{"4"}}{\text{"1","2","3","4","5","6"}} = \frac{1}{6}$$

Worksheet 14.3 (cont.)—Calculation of Independent Events

Two or More Events

You can calculate the probability of two or more Events by multipling the individual probabilities.

So, for Independent Events:

P(A and B) = P(A) x P(B)

Example: Probability of 3 Heads in a Row

For each toss of a coin a "Head" has a probability of 0.5 :

0.5

0.5 x 0.5 = 0.25

0.5 x 0.5 x 0.5 = 0.125

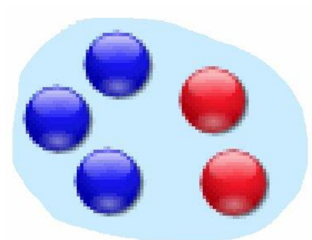
So the Probability of getting three Heads in a Row is 0.125.

Worksheet 14.3 (cont.)—Dependent Events

Dependent Events are affected by previous events.

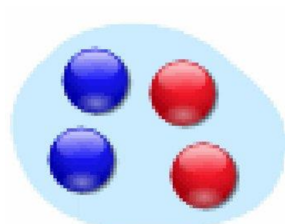
Example: Marbles in a Bag

There are 3 blue and 2 red marbles in a bag.
What is the probability of drawing a blue marble on the first and second draw?



P(Blue 1st Draw) $= \frac{3}{5}$

after the first draw you have changed the chances for the next draw



P(Blue 2nd Draw) $= \frac{2}{4} = \frac{1}{2}$

The probability of

P(Blue 1st Draw and Blue 2nd Draw) = P(Blue 1st Draw) x P(Blue 2nd Draw)

P(Blue 1st Draw and Blue 2nd Draw) $= \frac{3}{5} \times \frac{1}{2}$

P(Blue 1st Draw and Blue 2nd Draw) $= \frac{3}{10}$

Replacement

Note: if you had replaced the marbles in the bag each time, then the chances would not have changed and the events would be independent:

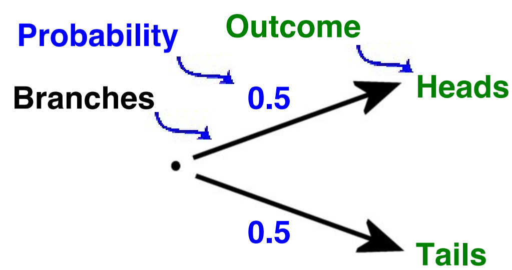
- With Replacement: the Events are Independent (the chances don't change)
- Without Replacement: the Events are Dependent (the chances change)

Handout 14.4—Tree Diagram Example

Probability Tree Diagrams

Calculating probability is confusing at times, especially for multiple events.
Tree diagrams give you a visual and more simple way to solve complex probability problems
The diagrams are composed of three items: Branches, Probabilites, and Outcomes.

Example: Probability of tossing a coin.

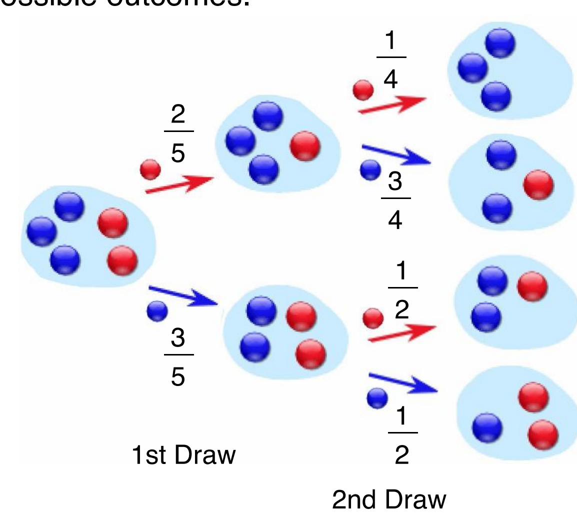


There are two Branches (Heads and Tails)

- The probability for each Branch is written on the Branch (0.5)
- The Outcome is written at the end of the Branch.

Now let us graph the previous example of the 3 blue and 2 red marbles in a bag.
What is the probability of drawing two red marbles from the bag with no replacements?

Graph out all possible outcomes.



	Outcomes
----> $\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$	Red, Red
----> $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$	Red, Blue
----> $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$	Blue, Red
----> $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$	Blue, Blue

You may find the probability of any outcome by multiplying the probabilities along any path.