Weekly Focus: solids
Weekly Skill: volume, surface area

LESSON 48: Pyramids, Cones, and Spheres

Lesson Summary: For the warm up, students will solve a problem about Lake Superior. In Activity 1, they will calculate the volume and surface area of spheres. In Activity 2, students will calculate the volume of pyramids and cones. In Activity 3, students they solve for the surface area of pyramids and cones. In Activity 4, they will do word problems. Activity 5 is an application problem about a big sinkhole that occurred in Guatemala. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 48:

- The geometry notes come from: http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf (pages 22 – 26)
- Video A (length 5:15) on volume of pyramids, cones, and spheres
  Video B (length 8:30) on surface area of pyramids, cones, and spheres
  The videos are required for teachers and optional for students
- 3 Worksheets (48.1, 48.2, 48.3) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 108 – 109)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 154 – 157)
- The application activity comes from the site: http://robertkaplinsky.com/work/sink-hole/

Objectives: Students will be able to:

- Solve the word problem about Lake Superior
- Calculate the volume and surface area of spheres, cones, and pyramids
- Solve word problems about these solids
- Solve a real-life problem about a sinkhole that happened in Guatemala

ACES Skills Addressed: N, CT, LS, EC
CCRS Mathematical Practices Addressed: Model with Math, Mathematical Fluency, Use Tools Strategically
Levels of Knowing Math Addressed: Intuitive, Pictorial, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 48 Warm-up: Solve the Lake Superior Questions	Time: 5-10 Minutes

Write on the board: Lake Superior is located between Canada and the U.S. There are about 175,000 Canadians living along the lake's northern border and about 425,000 Americans along its southern border.

Basic Questions:

- What is the ratio of Canadians to Americans living along Lake Superior?
  - $\frac{175,000}{425,000} = \frac{7}{17}$
- Americans consist of what percent of those living along Lake Superior?
  - $\frac{425,000}{600,000} = \frac{425}{600} = 71\%$

Extension Question:

- The volume of water in Lake Superior is 3.0×10^{15} gallons. Write this number in standard form.
  - 3,000,000,000,000,000 (3 quadrillion gallons)
  - Note: This enough water to cover all of South and North America in one foot of water!

Lesson 48 Activity 1: Volume and Surface Area of a Sphere	Time: 15 Minutes

1. Use the attached **Notes, pages 22 – 26,** for reference.
2. A sphere is a round figure with no bases or faces; a ball is a good example.
3. The formulas for volume and surface area of a sphere are similar to each other. See below.
4. Solve the volume and SA of the spheres on **Worksheet 48.1.** Do #1 and #9 as examples. If the diameter is given, it must be divided by two to get the radius.
5. Students can do some of the problems in class and complete the rest for homework.

Surface area and volume of a:

rectangular/right prism	SA = $ph + 2B$	V = Bh
cylinder	SA = $2\pi rh + 2\pi r^2$	V = $\pi r^2 h$
pyramid	SA = $\frac{1}{2}ps + B$	V = $\frac{1}{3}Bh$
cone	SA = $\pi rs + \pi r^2$	V = $\frac{1}{3}\pi r^2 h$
sphere	SA = $4\pi r^2$	V = $\frac{4}{3}\pi r^3$
	(p = perimeter of base with area B; $\pi \approx 3.14$)	

Lesson 48 Activity 2: Volume of Pyramids and Cones	Time: 20 Minutes

1. The objective of this activity is to find the volume of cones and pyramids.
2. These two are grouped together because their formulas are similar. For both, the volume = 1/3 area of base x height.
3. A **cone** is a solid 3-dimensional object with a circular base and one vertex (point). A good example is an ice cream cone (without the ice cream in it).
4. Since a cone has a circle as the base, we use 1/3 times πr^2 for the area of the base and multiply it by the height.
5. A **pyramid** is a solid 3-dimensional object with a polygon base and triangular sides that meet at an apex (point). A pyramid can be a square pyramid, a triangular pyramid, a pentagonal pyramid, etc.
6. The formula for the volume of a pyramid is 1/3 **B**, the area of its base, multiplied by its height.
7. Do #1 and #2 on **Worksheet 48.2** as examples. Then students can do the rest on their own.
8. Volunteers can do a few problems on the board.

Lesson 48 Activity 3: Surface Area of Pyramids and Cones	Time: 20 Minutes

1. The objective of this activity is to calculate the SA of pyramids and cones.
2. The **lateral area**, as referred to in videos and worksheets, means the sum of the area of all surfaces except for the top and bottom bases.
3. The **slant height**, referred to as ***s*** or ***l*** on videos or worksheets, is the height of the face, or the height of the triangles on the pyramid. The ***s*** is used on the GED formula sheet.
4. Finding the surface area of the pyramid means finding the area of each triangle and adding the area of the base.
5. Use #10 on **Worksheet 48.3** as an example. Have the students find the area of one triangle, multiply it by 4, and add the area of the base. They should get ½ (4)(6.3)(4) + 16 = 66.4.
6. The formula given on the GED test gets the same results and is a little shorter, but it is important for students to understand what they are doing when they calculate the surface area.
7. The SA of a cone is $\pi r s + \pi r^2$ = π multiplied by the radius by the slant height of the cone + the area of the circular base.
8. Use #1 on **Worksheet 48.3** as an example. The radius is 13 and the slant height is 30. Using the formula, we get $(3.14)(13)(30) + (3.14)(13^2) = 1224.6 + 530.66 = 1755.26$ units^2.
9. Questions #1-6 on the worksheet are meant to label the parts of the figures correctly, and questions #7-12 are to solve. You can have students solve all of them for SA if you have time.
10. Have volunteers do 1-2 problems on the board.

Lesson 48 Activity 4: Word Problems	Time: 45 Minutes

1. Do the problems on **pages 108-109** of the **student book** together.
2. Have students do the problems in the **workbook pages 154-157** independently.
3. Volunteers can solve a few of the more challenging problems on the board.

Lesson 48 Application: Guatemalan Sinkhole	Time: 15 Minutes

1. Become familiar with the activity before class.
2. Show the students the photo of the sinkhole, which will foster good discussion.
3. Follow the suggested format of asking questions and discussion as time permits.
4. The purpose of an activity such as this one is for students to see there are real life applications of the formulas and to make it interesting.

Lesson 48 Notes on Spheres, Cones, and Pyramids

Our next set of formulas is going to be for spheres. A sphere is most easily thought of as a ball. The official definition of a sphere is a three-dimensional surface, all points of which are equidistant from a fixed point called the center of the sphere. A circle that runs along the surface of a sphere to that it cuts the sphere into two equal halves is called a great circle of that sphere. A great circle of a sphere would have a diameter that is equal to the diameter of the sphere.

Surface Area of a Sphere

$$SA = 4pr^2$$

r = the radius of the sphere
π = the number that is approximated by 3.141593

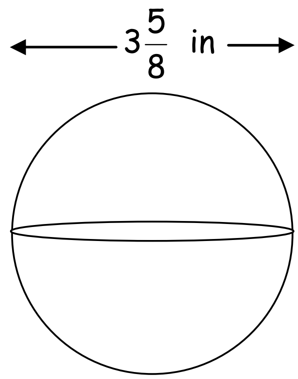
Volume of a Sphere

$$V = \frac{4}{3}pr^3$$

r = the radius of the sphere
π = the number that is approximated by 3.141593

Example 3:

Find the volume and surface area of the figure below



Solution:

This is a sphere. We are given that the diameter of the sphere is $3\frac{5}{8}$ inches. We need to calculate the radius of the sphere to calculate the volume and surface area. The radius of a sphere is half of its diameter. This means that the radius is $r = \frac{1}{2}d = \frac{1}{2}\left(3\frac{5}{8}\right) = \frac{1}{2}\left(\frac{29}{8}\right) = \frac{29}{16} = 1.8125$ inches. We can now just plug this number in to the formulas to calculate the volume and surface area.

Volume: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.8125^3\right) \approx 24.941505$ cubic inches.

Surface Area: $SA = 4\pi r^2 = 4\pi\left(1.8125^2\right) \approx 41.28219$ square inches.

Our next formulas will be for finding the volume of a cone or a pyramid. These two formulas are grouped together since they are very similar. Each is basically 1/3 times the area of the base of the solid times the height of the solid. In the case of the cone, the base is a circle. In the case of the pyramid, we will have a base that is a rectangle. The height in both cases is the perpendicular distance from the apex to the plane which contains the base.

A pyramid is a solid figure with a polygonal base (in our case a rectangle) and triangular faces that meet at a common point (the apex). A cone is the surface of a conic solid whose base is a circle. This is more easily thought of as a pointed ice-cream cone whose top is circular and level.

Volume of a Rectangular Pyramid

$$V = \frac{1}{3}lwh$$

l = the length of the base of the pyramid
w = the width of the base of the pyramid
h = the perpendicular height of the pyramid

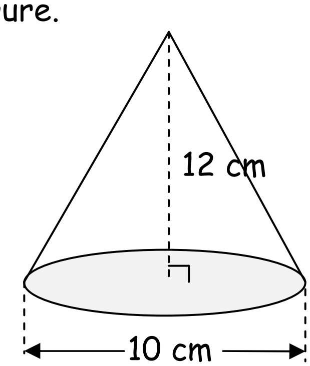
Volume of a Cone

$$V = \frac{1}{3}pr^2h$$

r = the radius of the circular base of the cone
π = the number that is approximated by 3.141593
h = the perpendicular height of the cone

Example 5:

Find the volume of the figure.

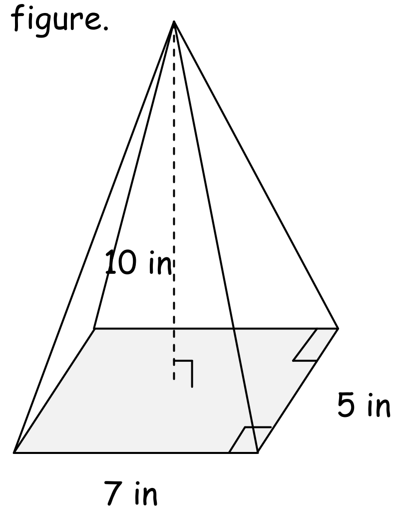


Solution:

Since the figure has a circular base and looks like an ice cream cone, this must be a cone. In order to find the volume of a cone, we need the radius of the circular base and the height (perpendicular height) of the cone. The height is given as 12 centimeters. The other measurement of 10 centimeters is the diameter of the circular base. We thus must calculate the radius to get $r = \frac{1}{2}d = \frac{1}{2}(10) = 5$ centimeters. We are now ready to plug into the volume of a cone formula. $V = \frac{1}{3}pr^2h = \frac{1}{3}p\left(5^2\right)(12) = 100p$ cubic centimeters is the exact volume. An approximation of this volume would be 314.159266 cubic centimeters.

Example 6:

Find the volume of the figure.

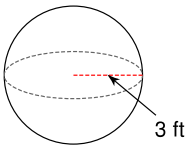


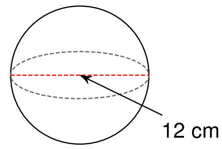
Solution:

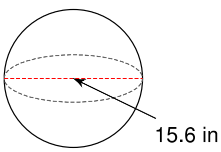
The base of this figure is a rectangle and the sides of the figure are triangles, thus this figure is a rectangular pyramid. The height (perpendicular height) is 10 inches. The length of the base is 7 inches, and the width of the base is 5 inches. Since we have all of the parts for the volume formula, we can just plug into the volume of a rctangular pyramid formula to get $V = \frac{1}{3}lwh = \frac{1}{3}(7)(5)(10) = \frac{350}{3}$ cubic inches. An approximation of this volume would be 116.66667 cubic inches.

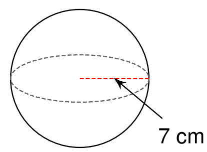
Worksheet 48.1 Spheres

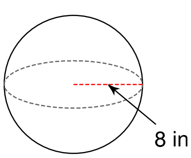
Find the surface area of each figure. Round your answers to the nearest tenth, if necessary.

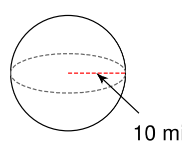
1) 

2) 

3) 

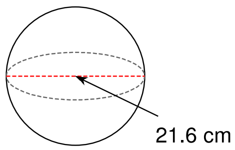
4) 

5) 

6) 

Find the volume of each figure. Round your answers to the nearest tenth, if necessary.

9)



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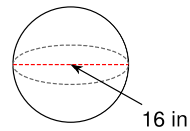

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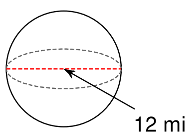

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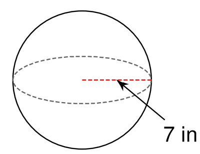

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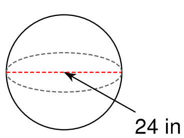

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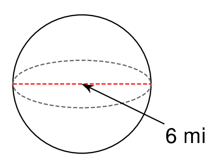
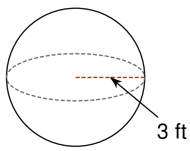


Worksheet 48.1 Answers

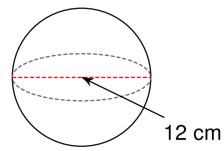
Find the surface area of each figure. Round your answers to the nearest tenth, if necessar

1)



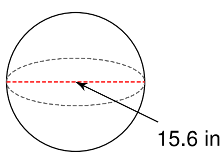
113.1 ft^2

2)



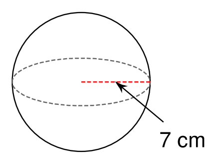
452.4 cm^2

3)



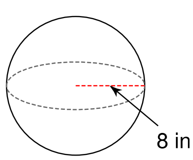
764.5 in^2

4)



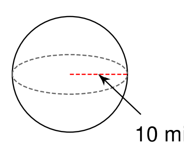
615.8 cm^2

5)



804.2 in^2

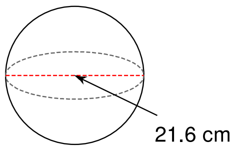
6)



1256.6 mi^2

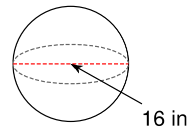
Find the volume of each figure. Round your answers to the nearest tenth, if necessary.

9)



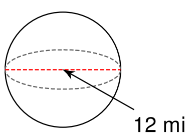
$5276.7\ cm^3$

10)



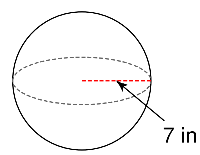
$2144.7\ in^3$

11)



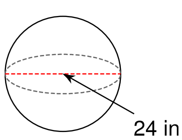
$904.8\ mi^3$

12)



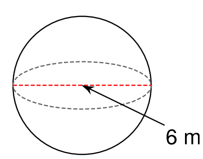
$1436.8\ in^3$

13)



$7238.2\ in^3$

14)

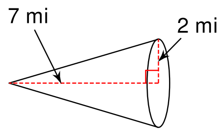


$113.1\ mi^3$

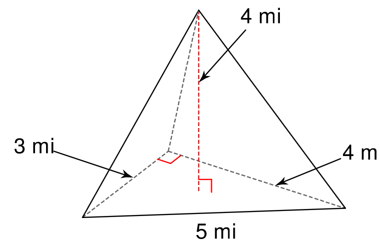
Worksheet 48.2 Volume of Pyramids and Cones

Find the volume of each figure. Round your answers to the nearest tenth, if necessary.

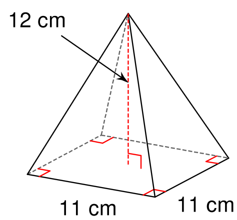
1)



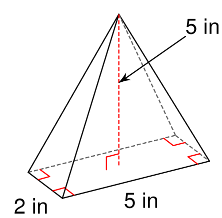
2)



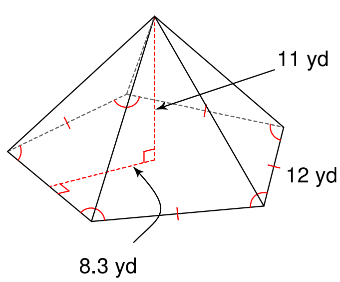
3)



4)



5)



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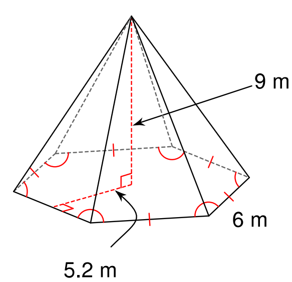
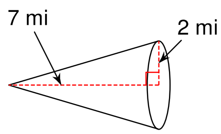
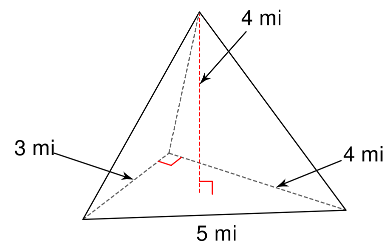


Worksheet 48.2 Answers

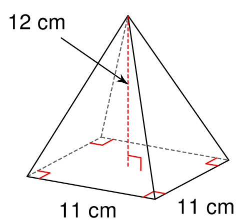
Find the volume of each figure. Round your answers to the nearest tenth, if necessary.

1)


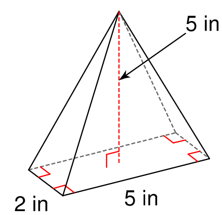
29.3 mi^3

2)


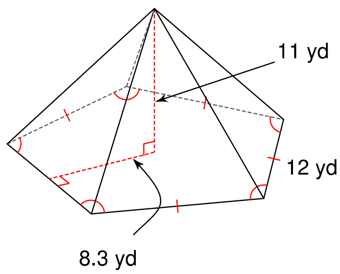
8 mi^3

3)


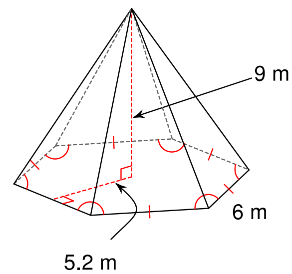
484 cm^3

4)


16.7 in^3

5)


913 yd^3

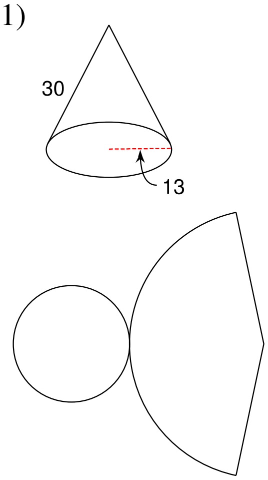
6)


280.8 m^3

Worksheet 48.3 Surface Area of Pyramids and Cones

Copy the measurements given onto the net of each solid.

1)



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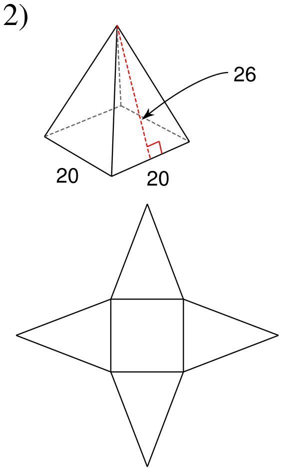
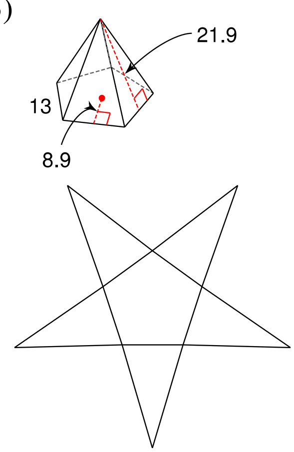
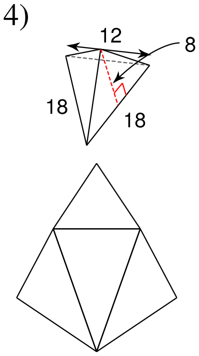


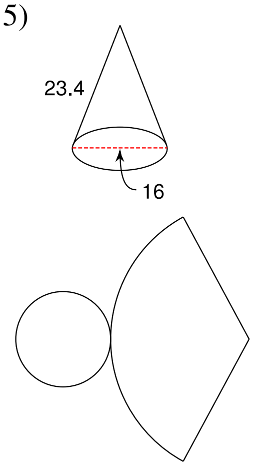
3)



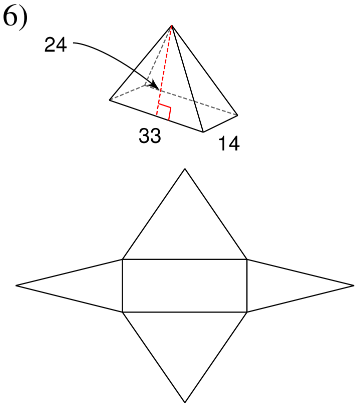
4)



5)

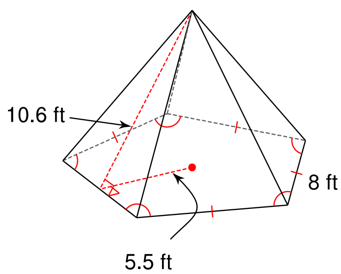


6)

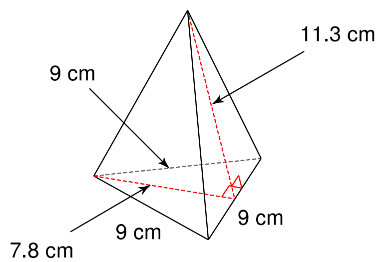


Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

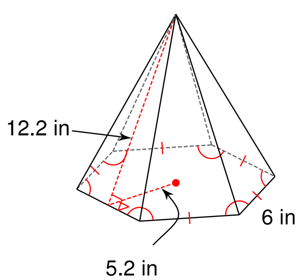
7)



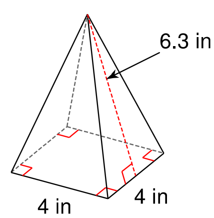
8)



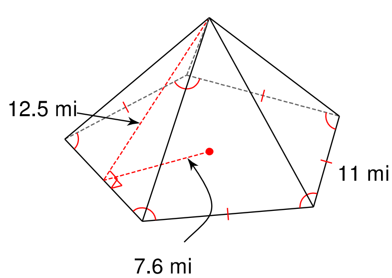
9)



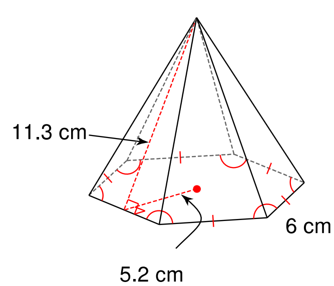
10)



11)



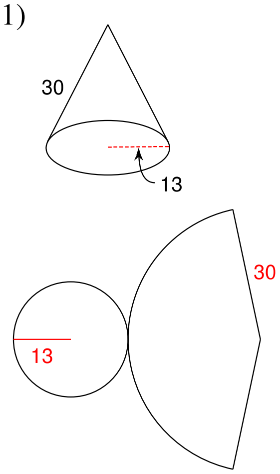
12)



Worksheet 48.3 Answers

Copy the measurements given onto the net of each solid.

1)



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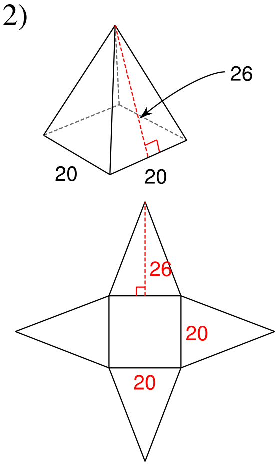
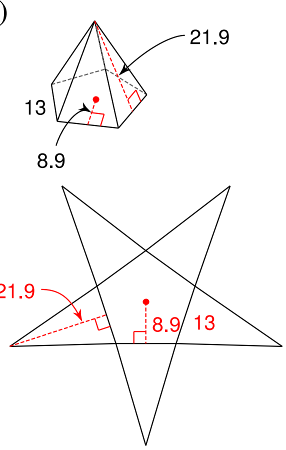
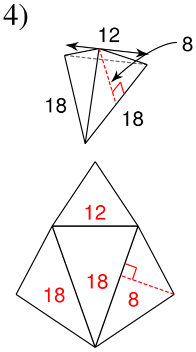


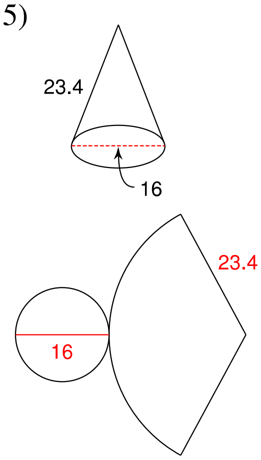
3)



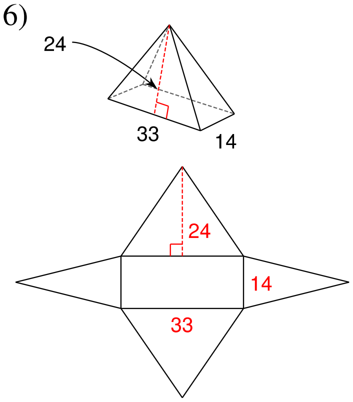
4)



5)

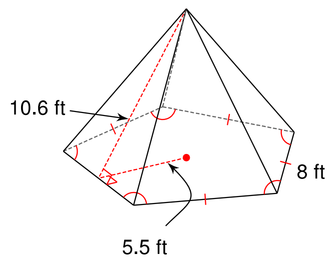


6)



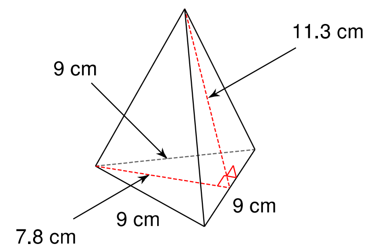
Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

7)



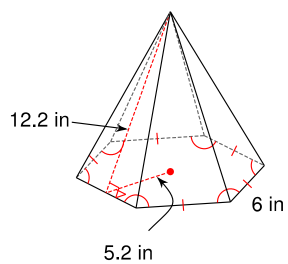
212 ft²; 322 ft²

8)



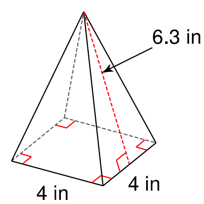
152.6 cm²; 187.7 cm²

9)



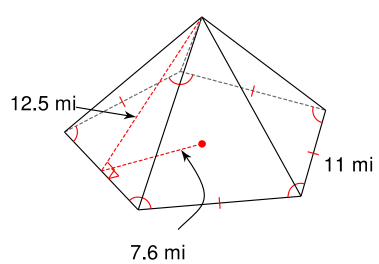
219.6 in²; 313.2 in²

10)



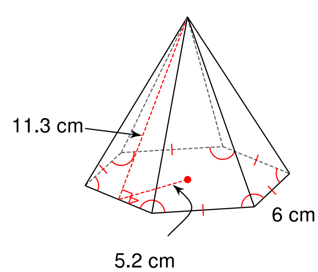
50.4 in²; 66.4 in²

11)



343.8 mi²; 552.8 mi²

12)



203.4 cm²; 297 cm²

Lesson 48 Application: The Guatemalan Sinkhole

robertkaplinsky.com http://robertkaplinsky.com/work/sink-hole/

How Big Is The 2010 Guatemalan Sinkhole?

The Situation

In 2010, an enormous sinkhole (pictured below) suddenly appeared in the middle of a Guatemalan neighborhood and swallowed a three-story building above it.

The Challenge(s)

- How much material will they need to fill the sinkhole?
- How much will it cost to fill the sinkhole in?

Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What is a guess that is too low?
- What is a guess that is too high?
- What is your best guess?

Consider This

The first thing a person should think when they see this photo is “There is no way this is real!” or “This is totally PhotoShopped!” That is what makes the photo so amazing because it is completely real and documented by many reputable sources such as National Geographic, Time Magazine, and CNN. Each of these articles can explain why and how the sinkhole (technically called a “piping pseudokarst“) happened better than I can, so I will focus on the math involved.

One of the sinkhole’s amazing properties is how incredibly cylindrical it is. The other thing I find striking is how deep it appears. It looks like it goes down and never stops. I wouldn’t be surprised if students thought it might go all the way through the Earth and come out on the other side (which apparently is not China but actually in the middle of the Indian Ocean). In actuality it does stop and this video below shows that better.

All the challenges revolve around knowing the sinkhole’s dimensions, however its reported dimensions vary. You can have students discuss the three reported options and debate to find the one that is most reasonable or skip this step and go with CNN article I have included (which I believe has the most accurate dimensions):

- National Geographic: “60 feet (18 meters) wide and about 30 stories deep” (assuming a story is about 10 feet, then about 300 feet deep)
- Time Magazine: “runs some 200 ft. deep”
- CNN: “The 20-meter (about 66 feet) diameter sinkhole is about 30 meters (about 100 feet) deep.”
- Slate: “A sinkhole, 65 feet across and 100 feet deep”

I think it is worth having students reflect on how reasonable these estimates are. Here are some of my thoughts:

- National Geographic, CNN, and Slate seem to agree that the diameter is about 20 meters, so that is what I will assume.
- The depth measurement varies from 100 feet to 200 feet to 300 feet. Obviously that is a huge difference.
- The pictures don't seem to be helpful in determining the depth.
- Looking at the video, the sinkhole looks to be a little more than 1.5 times deeper than it is wide. So, that seems to match up with the CNN and Slate measurements.

Going forward with dimensions of diameter of 66 feet and depth of 100 feet, we can use the formula for the volume of a cylinder: pi * radius^2 * height. That gives us pi * 33^2 & 100 cubic feet or approximately 342,119.45 cubic feet.

Interestingly, Slate appears to have made a mathematical error in their reporting (below)

> It's not clear whether cement is the best option, however. A 6,500-cubic-foot wad of concrete may serve to concentrate water runoff in other areas, leading to more sinkholes.

The only way I see how they could get 6,500 cubic feet as an answer using their reported dimensions of 65 feet in diameter and 100 feet in depth is to multiply 65 by 100. Clearly that is not correct. This is a great opportunity for students to double check the validity of their answer and defend their reasoning.

Update: I actually contacted the article's author and he replied "Apparently you picked the wrong article for a math lesson! I apologize. It appears you are correct." So, it is an example of how even professionals make silly mistakes and how we have to check our work to make sure it is reasonable. Using low, high, and best guesses is helpful for this.

The next challenge is "How much will it cost to fill the sinkhole in?". Obviously a lot of assumptions need to be made. Slate reported that in 2007 Guatemala filled a larger sinkhole (about 330 feet deep) with concrete. So, using that information:

A cubic foot of concrete currently costs about $4. $4 * 342,119.45 costs about $1,368,477.80. That obviously does not include labor, material transportation, or really anything besides the concrete itself. The price of concrete also varies, so it would be reasonable for the total cost to be somewhere between $1 and $2 million. In the Slate example the previous sinkhole cost $2.7 million so this seems like a reasonable estimate.

- View of the sinkhole with the helicopter's shadow in the hole.