Weekly Focus: prisms, cylinders
Weekly Skill: calculate area and volume

LESSON 47: Prisms and Cylinders

Lesson Summary: For the warm up, students will solve a problem about the earth and the moon. In Activity 1, students will find the surface area and volume of a prism. In Activity 2, they will do the same for a cylinder. Activity 3 is a worksheet to practice the computations. Activity 4 is the word problems in the student book and workbook. Activity 5 is a real-life application question. There is also an optional extra activity for homework. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 47:

- Boxes, cylinders (cans or oatmeal containers are good), rulers, scissors, and paper
- Notes on volume and surface area of cylinders and prisms. The geometry notes come from this site: http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf (pages 18-22)
- Video A (length 3:50) on volume of a prism
  Video B (length 10:30) on finding surface area of prisms
  Video C (length 5:20) on surface area and volume of cylinders
  The videos are required for teachers and recommended for students
- 1 Worksheet (47.1) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 106 – 107)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 150 – 153)
- 1 Application Activity (attached)
- 1 Extra Activity for Homework or Extra Time

Objectives: Students will be able to:

- Solve the earth and moon dimensions word problem
- Find the volume and surface areas of a cylinder (can) and a prism (box)
- Calculate the volume and surface areas and solve word problems
- Do a real-life application activity about pouring a concrete driveway.

ACES Skills Addressed: N, CT, LS, ALS
CCRS Mathematical Practices Addressed: Building Solution Pathways, Mathematical Fluency, Model with Math
Levels of Knowing Math Addressed: Intuitive, Pictorial, Concrete, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 47 Warm-up: Solve the earth and moon problem	Time: 10 Minutes

Write on the board: The circumference of the earth is 40,075 km. The circumference of the moon is about 27% that of the earth.

Basic Questions:

- What is the circumference of the moon in km?
  - (0.27)(40,075) = 10,820 km
- What is the circumference of the moon in miles?
  - Hint if students need it: 0.6 x km = miles
  - (0.6)(10,820) = 6,492 miles. C of moon is about 6,500 miles.
- What is the circumference of the earth in miles?
  - (0.6)(40,075) = 24,045 miles

Extension Questions:

- If the earth's diameter is 12,750 km, what is its radius?
  - Half of the diameter is the radius = 6,375 km
- If the moon's radius is 1738 km, what is its diameter?
  - Radius x 2 = diameter = 3,476 km
- Note to teacher: The above are all estimates. If you look up the dimensions of the earth and the moon online, you will find more precise measurements.

Lesson 47 Activity 1: Volume and Surface Area of Prisms	Time: 15 Minutes

1. The objective of Activity 1 is to learn how to find the volume and the surface area of a prism.
2. A **prism is a 3-dimensional figure with 2 congruent bases.** A good example is a box; the two ends are the bases.
3. Give students boxes and rulers. Have students measure the area of each surface. They will realize they only need to measure the area of the 3 different sides and double those to get the surface area of the whole box.
4. If you have time or someone needs more of a visual, have the students cut out sheets of paper that fit each of the 6 surfaces. The total surface area is the area of the top and bottom, the two ends (bases), and the two sides all combined.
5. Students have discovered the formula for surface area, which is the same as in the attached **Lesson 47 Notes, pages 18 – 22,** where the surface area formula is given as 2(lw + lh + wh).

6. The surface area as given on the formula sheet of the GED test is shorter. It is **SA = ph + 2B** where p=perimeter of a base, h= height, and B= area of the base.
7. Students can use the shorter formula once they understand the meaning of surface area.
8. Notes:
   a. As an alternative (if short on time), give the students the formula without taking time to have them discover it for themselves.
   b. The video and worksheet below refer to the **lateral area**, which is ph = perimeter of the base x height of prism
   c. Remind students that area is always squared.
   d. Use the **Notes, pages 18 – 22,** as a teaching guide or copy them for the students if they are helpful.
9. Now students can measure the volume of the box. The **volume** is how much space is occupied by an object.
10. Use **B**, the area of the base (one of the ends), and multiply it by the height.
11. Use Example 1 in the notes as another practice example. Give the students the measurements and have them use their GED formula sheet (a portion of it is given below) to solve for the surface area and the volume of the box.

Surface area and volume of a:

rectangular/right prism	SA = $ph + 2B$	V = Bh
cylinder	SA = $2\pi rh + 2\pi r^2$	V = $\pi r^2 h$
pyramid	SA = $\frac{1}{2} ps + B$	V = $\frac{1}{3} Bh$
cone	SA = $\pi rs + \pi r^2$	V = $\frac{1}{3} \pi r^2 h$
sphere	SA = $4\pi r^2$	V = $\frac{4}{3} \pi r^3$
	(p = perimeter of base with area B; $\pi \approx 3.14$)	

Lesson 47 Activity 2: Volume and Surface Area of Cylinders	Time: 15 Minutes

1. Give students a can or other type of cylinder, paper, ruler, and scissors.
2. Have them find the area of the top or bottom (a circle). They may remember the formula of a circle as **πr^2.**
3. To find the surface area of the outside of the can, have them cut out a piece of paper that will wrap around the can and then measure its area (width x length). This area around the can is called the lateral area, as described in the video.
4. The area of that sheet of paper should measure about the same as the lateral area given in the formula: **$2 \pi r h$**
5. The formula for SA of a cylinder is the area of two bases (top and bottom) of cans added to the lateral area. The formula is **$2 \pi r h + \pi r^2$.**
6. The **volume** of a cylinder is measured as **$\pi r^2 h$**, which is the area of the base times the height.
7. Have students practice measuring volume with the cans or cylinders they have.
8. Have students do example 2 from the notes as another practice.

Lesson 47 Activity 3: Practice Calculations	Time: 15 Minutes

1. Hand out **Worksheet 47.1.**
2. Questions 1 to 6 are to practice labeling the different parts of the prisms or cylinders.
3. Questions 7 to 12 are to solve for surface area.

Lesson 47 Activity 4: Word Problems	Time: 45 Minutes

1. Do the problems in the **student book pages 106-107** together.
2. Have students work independently in the **workbook pages 150 to 153**.
3. Do any of the challenging problems on the board if there is time.

Lesson 47 Application A: Pouring a Concrete Driveway	Time: 20 Minutes

Give students the problem:

Mike Jones bought an older house and wants to put in a new concrete driveway. The driveway will be 30 feet long, 10 feet wide, and 9 inches thick. Concrete is measured by the cubic yard. One sack of dry cement mix costs $7.30, and it takes four sacks to mix up one cubic yard. How much will it cost Mike?

See the attached explanation and answer.

Extra Time? Finish Early? Make a Box	Time: 20 Minutes

1. Give students the directions for this extra activity or for homework.
2. It is helpful for those who learn kinesthetically by doing something.

Lesson 47 Notes: Volume and Surface Area of Prisms and Cylinders

The surface area of a figure is defined as the sum of the areas of the exposed sides of an object. A good way to think about this would be as the

area of the paper that it would take to cover the outside of an object without any overlap. In most of our examples, the exposed sides of our objects will polygons whose areas we learned how to find in the previous section. When we talk about the surface area of a sphere, we will need a completely new formula.

The volume of an object is the amount of three-dimensional space an object takes up. It can be thought of as the number of cubes that are one unit by one unit by one unit that it takes to fill up an object. Hopefully this idea of cubes will help you remember that the units for volume are cubic units.

Surface Area of a Rectangular Solid (Box)

$$SA = 2(lw + lh + wh)$$

l = length of the base of the solid
w = width of the base of the solid
h = height of the solid

Volume of a Solid with a Matching Base and Top

$$V = Ah$$

A= area of the base of the solid
h = height of the solid

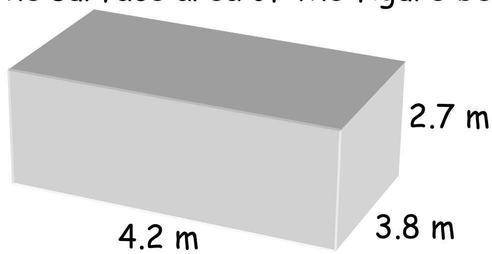
Volume of a Rectangular Solid
(specific type of solid with matching base and top)

$$V = lwh$$

l = length of the base of the solid
w = width of the base of the solid
h = height of the solid

Example 1:

Find the volume and the surface area of the figure below



Solution:

This figure is a box (officially called a rectangular prism). We are given the lengths of each of the length, width, and height of the box, thus we only need to plug into the formula. Based on the way our box is sitting, we can say that the length of the base is 4.2 m; the width of the base is 3.8 m; and the height of the solid is 2.7 m. Thus we can quickly find the volume of the box to be
$V = lwh = (4.2)(3.8)(2.7) = 43.092$ cubic meters.

Although there is a formula that we can use to find the surface area of this box, you should notice that each of the six faces (outside surfaces) of the box is a rectangle. Thus, the surface area is the sum of the areas of each of these surfaces, and each of these areas is fairly straight-forward to calculate. We will use the formula in the problem. It will give us
$SA = 2(lw + lh + wh) = 2(4.2 * 3.8 + 4.2 * 2.7 + 3.8 * 2.7) = 75.12$ square meters.

A cylinder is an object with straight sides and circular ends of the same size. The volume of a cylinder can be found in the same way you find the volume of a solid with a matching base and top. The surface area of a cylinder can be easily found when you realize that you have to find the area of the circular base and top and add that to the area of the sides. If you slice the side of the cylinder in a straight line from top to bottom and open it up, you will see that it makes a rectangle. The base of the rectangle is the circumference of the circular base, and the height of the rectangle is the height of the cylinder.

Volume of a Cylinder

$$V = Ah$$

A = the area of the base of the cylinder
h = the height of the cylinder

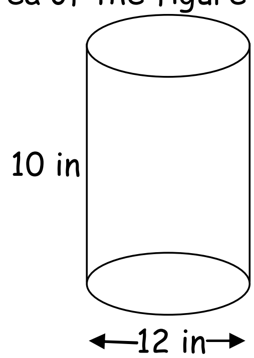
Surface Area of a Cylinder

$$SA = 2(pr^2) + 2prh$$

r = the radius of the circular base of the cylinder
h = the height of the cylinder
π = the number that is approximated by 3.141593

Example 2:

Find the volume and surface area of the figure below



Solution:

This figure is a cylinder. The diameter of its circular base is 12 inches. This means that the radius of the circular base is $r = \frac{1}{2}d = \frac{1}{2}(12) = 6$ inches. The height of the cylinderi s 10 inches.

To calculate the volume and surface area, we simply need to plug into the formulas.

Surface Area:

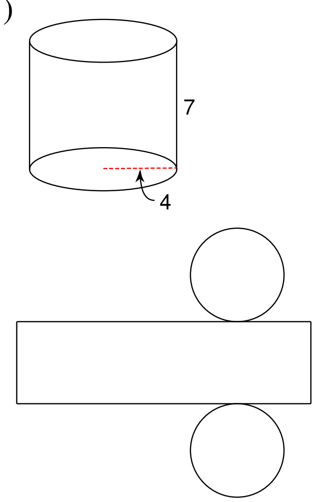
$SA = 2(pr^2) + 2prh = 2(p \times 6^2) + 2p(6)(10) = 72p + 120p = 192p$ square units. This is an exact answer. An approximate answer is 603.18579 square units.

Volume:

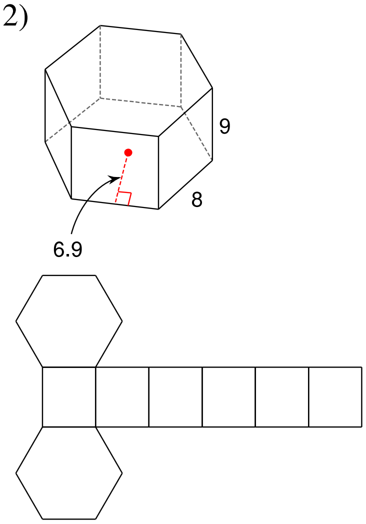
In order to plug into the formula, we need to recall how to find the area of a circle (the base of the cylinder is a circle). We will the replace A in the formula with the formula for the area of a circle. $V = Ah = \pi r^2 h = \pi(6^2)(10) = 360\pi$ cubic inches. An approximation of this exact answer would be 1130.97336 cubic inches.

Worksheet 47.1

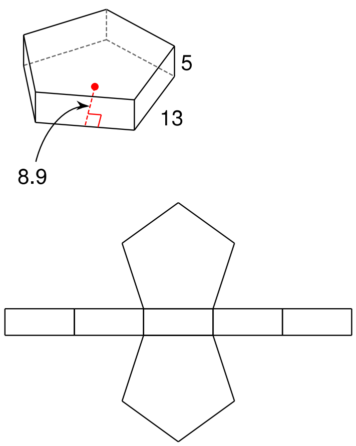
1)



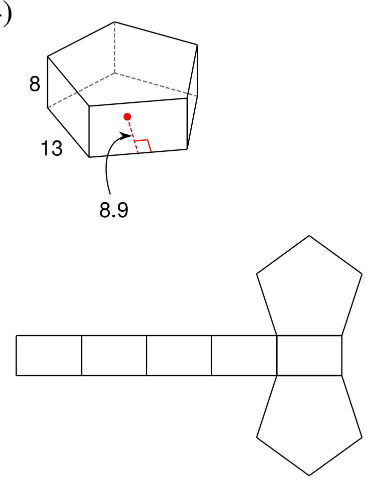
2)



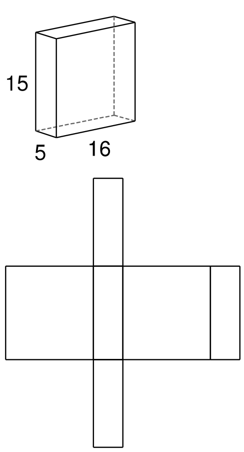
3)



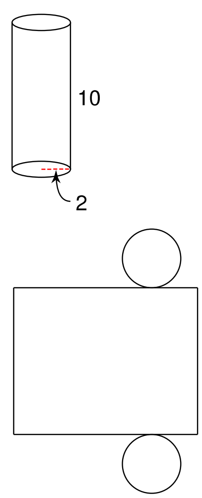
4)



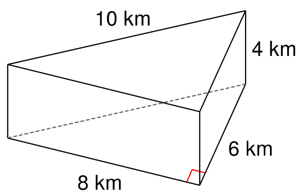
5)

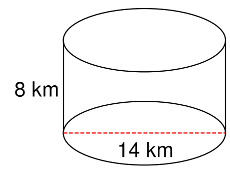


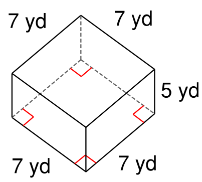
6)

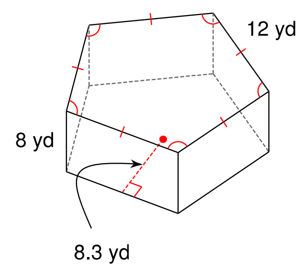


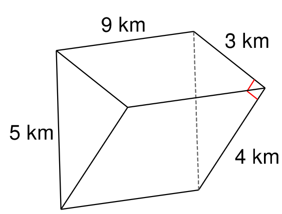
Find the lateral area and surface area of each figure. Round your answers to the nearest thousandth, if necessary.

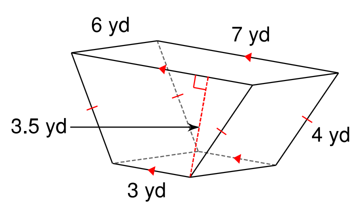
7) 

8) 

9) 

10) 

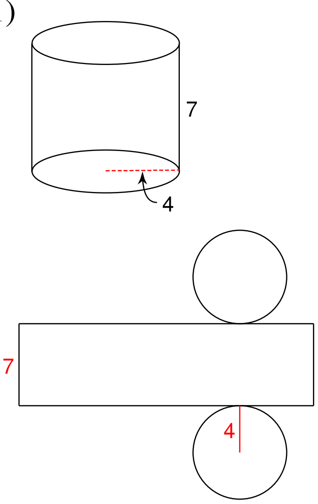
11) 

12) 

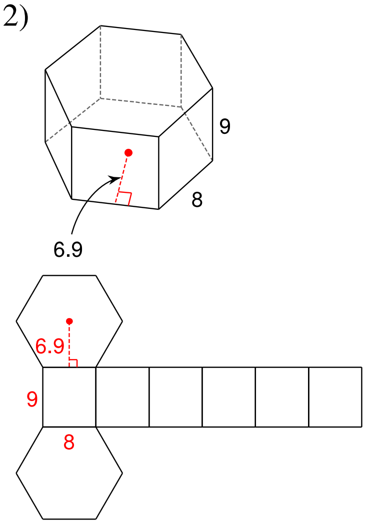
Worksheet 47.1 Answers

Copy the measurements given onto the net of each solid.

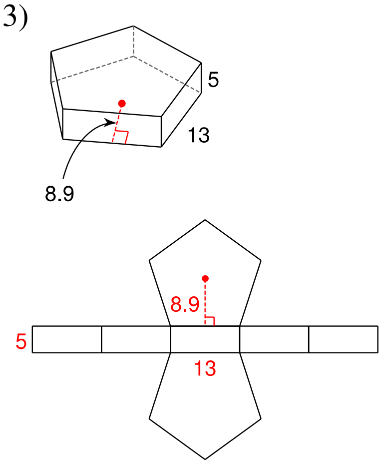
1)



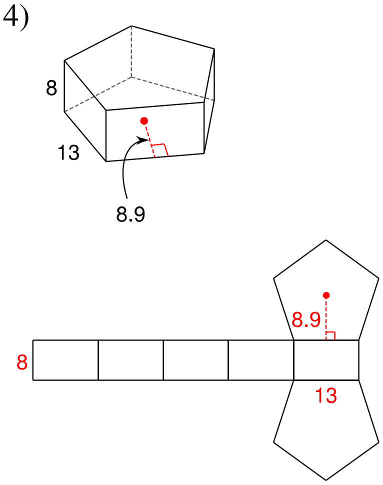
2)



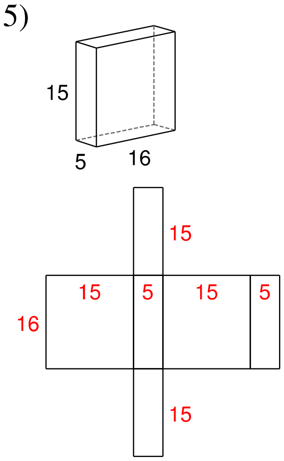
3)



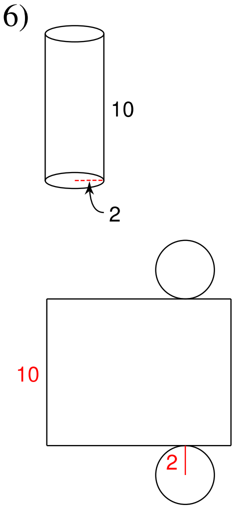
4)



5)

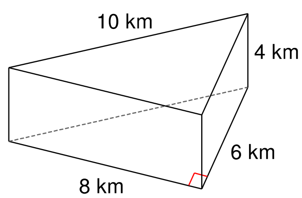


6)



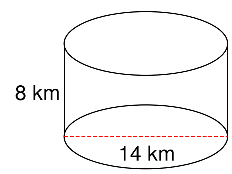
Find the lateral area and surface area of each figure. Round your answers to the nearest thousandth, if necessary.

7)



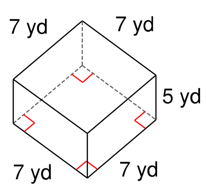
96 km²; 144 km²

8)



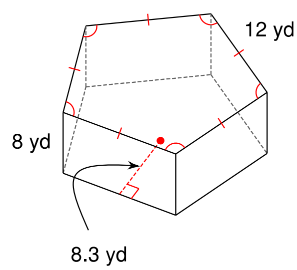
351.86 km²; 659.73 km²

9)



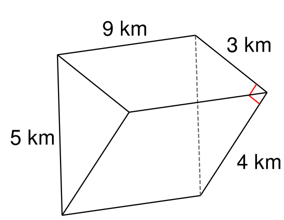
140 yd²; 238 yd²

10)



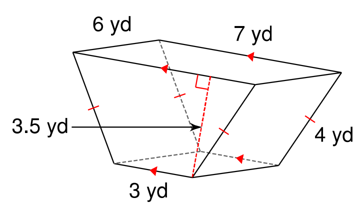
480 yd²; 978 yd²

11)



108 km²; 120 km²

12)



108 yd²; 143 yd²

Lesson 47 Application: Pouring a Concrete Driveway

Mike Jones bought an older house and wants to put in a new concrete driveway. The driveway will be 30 feet long, 10 feet wide, and 9 inches thick. Concrete (a mixture of sand, gravel, and cement) is measured by the cubic yard. One sack of dry cement mix costs \$7.30, and it takes four sacks to mix up 1 cubic yard of concrete. How much will it cost Mike to buy the cement?

Solution:

The driveway that is being poured will be a rectangular solid or box. Thus in order to answer this question, we will first need to find the volume of this box (the amount of cubic units it will take to fill this box). The problem tells us that concrete is measured by the cubic yard. This lets us know that those are the units we will want to calculate with. None of the dimensions of the driveway are given in yards. We will need to use our dimensional analysis (unit conversion) to convert all of the measurements to yards. This could be done at a later point, but this is the easiest place to take care of the conversion.

Convert length:

$$\frac{30 \text{ feet}}{1} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 10 \text{ yards}$$

Convert width:

$$\frac{10 \text{ feet}}{1} \times \frac{1 \text{ yard}}{3 \text{ feet}} = \frac{10}{3} \text{ yards}$$ We are not going to estimate this

value since the approximation with introduce error. As in the finance section, we don't want to round until the end of the problem or in a place where it is absolutely necessary.

Convert height:

$\frac{9 \text{ inches}}{1} \times \frac{1 \text{ yard}}{36 \text{ inches}} = \frac{1}{4}$ yards = .25 yards Since the decimal representation of this number is a terminating decimal, we can use this representation in our calculations.

Volume of driveway:

We are now ready to calculate the volume of the driveway.

$V = lwh = (10)\left(\frac{10}{3}\right)(.25) = \frac{25}{3}$ cubic yards.

Now, we are told that it takes 4 bags of cement to make one cubic yard of concrete. So we will now calculate how many bags of cement to buy. Mike's driveway is $\frac{25}{3}$ cubic yards and each of those cubic yards requires 4 bags of cement. Thus Mike will need $\frac{25}{3} \times 4 = \frac{100}{3}$ bags or approximately 33.333 bags. Here is where common sense needs to come in. There is no store that will sell .333 bags of cement mix. Stores only sell whole bags of cement mix. Thus we will need to round up to the next whole bag (we can't round down or we will not have enough cement mix to complete the driveway). This means that Mike will need to buy 34 bags of cement mix. Each of these bags will cost $7.30. This means that Mike will pay $7.30 \times 34 = \$248.20$ for the cement mix for this driveway.

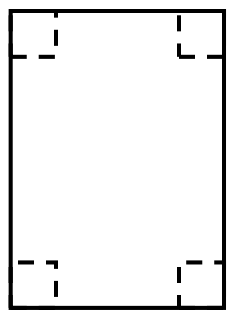
Activity: Extra Time? Finish Early?

Example 8:

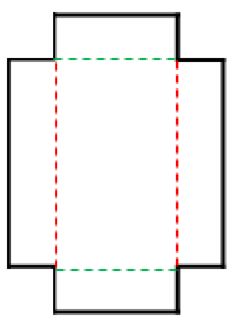
From an 8.5-inch by 11-inch piece of cardboard, 2-inch square corners are cut out and the resulting flaps are folded up to form an open box. Find the volume and surface area of the box.

Solution:

For this problem, it will be really helpful to make the box described above. You start with a standard piece of paper. You the cut out the dashed square indicated below from each corner. You make sure each side of the square is 2 inches in length.



What you are left with is the shape below.



You now fold up along the dashed lines to create a box. The box that we have created is a rectangular solid. This box has no top. Not having a top will not affect the volume of the box.

We only need to determine the length of the base of the box, the width of the base of the box, and the height of the box. The red dashed lines represent the length of the base of the box. The original length of the paper was 11 inches. We removed 2 inches from the top of the page and we also removed 2 inches from the bottom of the page. Thus the red dashed line is $11 - 2 - 2 = 7$ inches.

The green dashed line represents the width of the base of the box. The original width of the paper was 8.5 inches. We removed 2 inches from the left side of the page and also removed 2 inches from the right side of the page. Thus the green dashed line is $8.5 - 2 - 2 = 4.5$ inches.

We now need to think about the height of the box. Since we have folded up the sides for form the height of the box, we just need to determine how tall those sides are. Since they were made by cutting out 2-inch square from each corner, these sides must be 2 inches high.

Now we are ready of calculate the volumes
$V = lwh = (7)(4.5)(2) = 63$ cubic inches.

Now we need to calculate the surface area of our box. Since there is no top to this box, we can start formula for the surface area of a box. We will then need to subtract off the area of the top of the box. This will give us
$SA = 2(lw + wh + lh) = 2(7 \times 4.5 + 4.5 \times 2 + 7 \times 2) = 109$ square inches for the box with the top included. The top would have the same area as the base of the box. This would be $A = lw = (7)(4.5) = 31.5$ square inches. Thus the surface area of our figure is total surface area – area of the top = $109 - 31.5 = 77.5$ square inches.

There is another way to calculate the surface area of this box. The surface area is the amount of paper it would take to cover the box without overlap. You should notice that this is the same as the amount of paper we used to make the box. Thus, it is enough to calculate the area of the paper as shown here.