Weekly Focus: polygons, triangles
Weekly Skill: solve for missing measurement

LESSON 46: Scale Drawings

Lesson Summary: For the warm up, students will solve a rate problem. In Activity 1, students will solve for the missing measurement in similar figures. In Activity 2, they will do related word problems. There is an exit ticket and also an application activity at the end. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 46:

- Video (length 5:28) on similar figures. The video is required for teachers and optional for students.
- Rulers for the application activity
- Notes to copy and give to students, pages 37 – 42 (http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf)
- 1 Worksheet (46.1) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 104 – 105)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 146 – 149)
- Exit Ticket (https://www.yummymath.com/2013/ladder-figuring/)
- The application activity or its images can also be downloaded directly from the site: http://robertkaplinsky.com/work/what-should-the-freeway-sign-show/

Objectives: Students will be able to:

- Solve the rate word problem about grass seed
- Understand the relationship between similar figures
- Calculate the missing measurement by using proportions
- Solve 1 (exit ticket) or 2 (application activity) real-life problems

ACES Skills Addressed: N, CT, LS, EC
CCRS Mathematical Practices Addressed: Building Solution Pathways, Model with Math, Construct viable arguments and critique the reasoning of others
Levels of Knowing Math Addressed: Intuitive, Pictorial, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 46 Warm-up: Solve the rate problem	Time: 10 Minutes

Write on the board: Margaret has 2 lbs. of grass seed to spread on her front lawn. Each pound covers 400 ft^2. The instructions say to spread the seed at a rate of 4 oz. per 30 seconds.

Basic Question:

- How many minutes will it take to spread the 2 lbs. of seed?
  - 4 minutes. 2 lbs. = 32 oz. $\frac{4\ oz}{30\ seconds} = \frac{32\ oz}{240\ seconds}$. 240 seconds = 4 minutes

Extension Question:

- What is the radius of the largest circular lawn that Margaret can plant with her 2 pounds of seed?
  - A= 3.14 r^2. 800 = 3.14r^2. Solving for r gives us approximately16 feet.

Lesson 46 Activity 1: Similar Figures	Time: 30 Minutes

1. Copy the **Similar Triangles Notes, pages 37 – 42** for students. Use the notes as a basis for the examples.
2. The objectives of this lesson are to determine if **two figures are similar- they have different lengths of sides, but the lengths are proportional** – and to find missing numbers for similar figures.
3. Ask students when this might be used in real life. Responses may include the making of maps and in making blueprint models for construction projects. (You can also wait to ask this

question until after you have done one or two examples.)

4. Draw two triangles like on the first page of the notes. Explain that triangles with **congruent (equal) angles** will have opposite sides that are proportional. Sides opposite corresponding angles are corresponding sides.
5. We can find the missing side measurement by setting up proportions.
6. Do the 3 examples in the notes.
7. The relationship of other polygons works the same way.
8. Hand out **Worksheet 46.1**. Do the first problem together.
9. Explain that the **scale factor** is the ratio of one figure to another. An example is on a map, the legend tells you what the ratio on the map is compared to the actual distance.
10. For Question 7, the scale factor of A to B is 2 to 7. Set it up as a proportion of A/B = 2/7 = 6/X. Solving the proportion gives X = 21.

Lesson 46 Activity 2: Scale Drawings Word Problems	Time: 40 Minutes

1. Do the problems on **pages 104-105** in the **student book** together.
2. Have a few students do some of the problems on the board.
3. Now students can work independently on **pages 146-149** in the **workbook.**
4. Circulate to help.
5. Have volunteers do some of the more challenging problems on the board.

Lesson 46 Exit Ticket Option	Time: 5-10 Minutes

1. Show the picture of the large ladder at a festival (download the activity from the website). A copy is attached for reference.
2. Give students time to discuss and estimate how long it is. They can use the estimated length of cars and estimate how many car lengths it is.
3. *Answer:* length of ladder is 105 ft.

Lesson 46 Application: What Should Highway Sign Show?	Time: 25-30 Minutes

1. This activity includes the practice of scale drawings and of measurement in a real-life

context.

2. You will need rulers and be able to show the photos and the map on the application activity. You can download the activity directly from the website (link given above) if you prefer.
3. Discuss the situation by using some of the questions suggested in the lesson.
4. Become familiar with the lesson before presenting it to students.
5. There are two sets of question in the activity. Only do the first one if you don't have enough time.
6. As an alternative, you can tell the students the estimated measurements in cm to save time and have them solve the problem from there.

Lesson 46 Similar Triangles Notes

SIMILAR TRIANGLES

Objectives:

After completing this section, you should be able to do the following:

- Calculate the lengths of sides of similar triangles.
- Solve word problems involving similar triangles.

Vocabulary:

As you read, you should be looking for the following vocabulary words and their definitions:

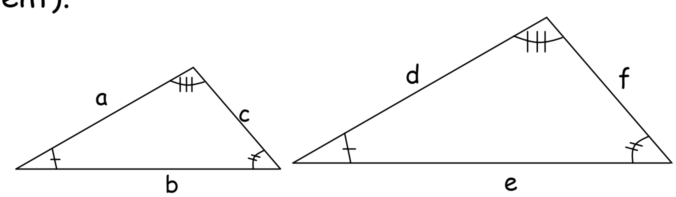
- similar triangles

Formulas:

You should be looking for the following formulas as you read:

- proportions for similar triangles

We will continue our study of geometry by looking at similar triangles. Two triangles are similar if their corresponding angles are congruent (or have the same measurement).



In the picture above, the corresponding angles are indicated in the two triangles by the same number of hash marks. In other words, the angle with one hash mark in the smaller triangle corresponds to the angle with one hash mark in the larger triangle. These angles have the same measure or are congruent.

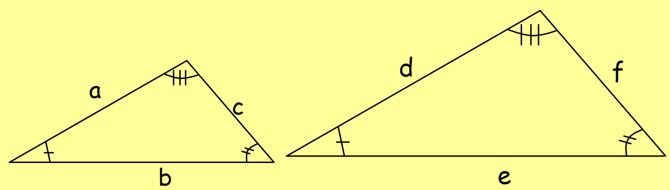
There are also corresponding sides in similar triangles. In the triangles above side a corresponds to side d (for example). These sides do not necessarily have the same measure. They do, however, form a ratio that is the same no matter which pair of corresponding sides the ratio is made from. Thus we can write the following equation

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

Notice that the sides of one particular triangle are always written on top of the fractions and the sides of the other triangle are always written on the bottom of the fractions. It does not matter which triangle is put in which part of the fraction as longs as we are consistent within a problem.

Similar Triangle Ratios

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

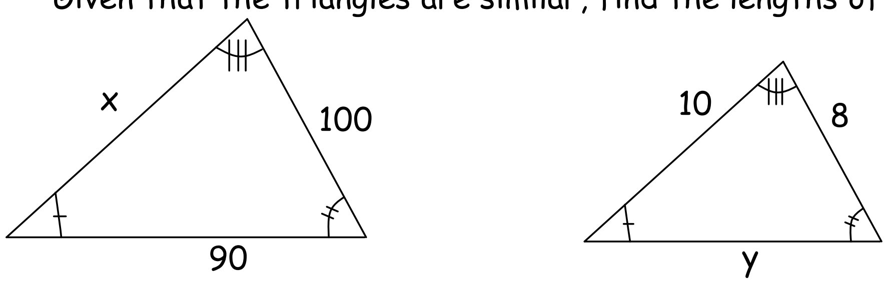


It should be noted that although our triangles are in the same relative position, this is not needed for triangles to be similar. One of the triangles can be rotated or reflected.

Please note that pictures below are not drawn to scale.

Example 1:

Given that the triangles are similar, find the lengths of the missing sides.



Solution:

There is one side missing in the triangle on the left. This side is labeled x. There is also one side missing from the triangle on the right. This side is labeled y. The side x in the triangle on the left corresponds to the side labeled 10 in the triangle on the right. We know this because these sides connect the angle with one hash mark to the angle with three hash marks in each of the triangles. The side 100 in the triangle on the left corresponds to the side labeled 8 in the triangle on the right. Finally the side 90 in the triangle on the left corresponds to the side labeled y in the triangle on the right. We will need this information to find values for x and y.

Find x:
We will start by finding the value for x. We will need to form a ratio with the side labeled x and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just need to solve the equation for x. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

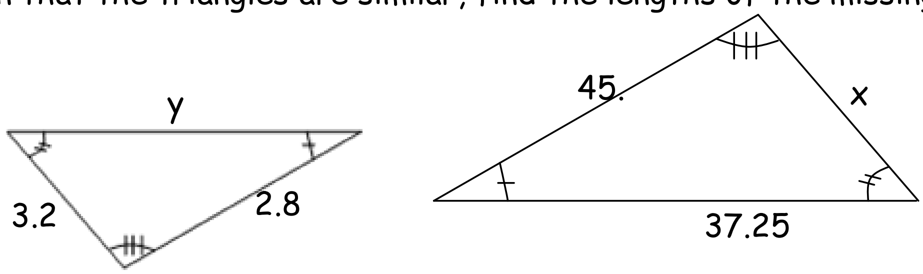
$$\frac{x}{10} = \frac{100}{8}$$
$$8(x) = 10(100)$$
$$8x = 1000$$
$$x = 125 \text{ units}$$

Find y:
We will continue by finding the value for y. We will need to form a ratio with the side labeled y and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Although we know the measurements of all the other sides of the triangle, it is best to avoid using a side we just calculate to make further calculations if possible. The reason for this is that if we have made a mistake in our previous calculation, we would end up with an incorrect answer here as well. Once we make an equation using these ratios, we will just need to solve the equation for y. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{90}{y} = \frac{100}{8}$$
$$8(90) = y(100)$$
$$720 = 100y$$
$$7.2 \text{ units} = y$$

Example 2:

Given that the triangles are similar, find the lengths of the missing sides.



Solution:

Our first job here is to determine which sides in the triangle on the left correspond to which sides in the triangle on the right. The side in the triangle on the left labeled y joins the angle with one hash mark and the angle with two has marks. The side in the triangle on the right that does this is the side labeled 37.25. Thus these are corresponding sides.

In a similar manner, we can see that the side labeled 3.2 in the triangle on the left corresponds to the side labeled x on the triangle on the right. Finally the side labeled 2.8 in the triangle on the left corresponds to the side labeled 45 in the triangle on the right.

Find x:

We will start by finding the value for x. We will need to form a ratio with the side labeled x and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just need to solve the equation for x. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{3.2}{x} = \frac{2.8}{45}$$
$$3.2(45) = x(2.8)$$
$$144 = 2.8x$$
$$\frac{144}{2.8}\text{ units} = x$$

This is approximately 51.42857 units.

Find y:

We will continue by finding the value for y. We will need to form a ratio with the side labeled y and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just need to solve the equation for y. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{y}{37.25} = \frac{2.8}{45}$$
$$y(45) = 37.25(2.8)$$
$$45y = 104.3$$
$$y = \frac{104.3}{45}\text{ units}$$

This is approximately 3.21778 units.

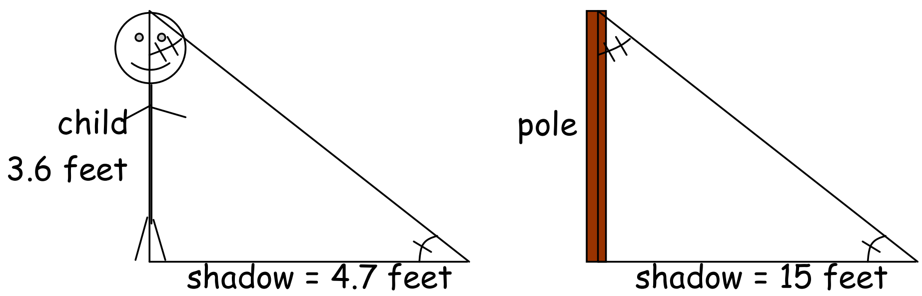
Our final example is an application of similar triangles

Example 3:

A 3.6-foot –tall child casts a shadow of 4.7 feet at the same instant that a telephone pole casts a shadow of 15 feet. How tall is the telephone pole?

Solution:

For this problem, it helps to have a picture.



When we look at the picture above, we see that we have two triangles that are similar. We are looking for the height of the pole. The pole side of the triangle on the right corresponds to the side of the child side of the triangle on the left. The two shadow sides of the triangles also correspond to each other. Thus we can write an equation of ratios to find the height of the pole.

$$\frac{3.6}{\text{pole}} = \frac{4.7}{15}$$

$$(3.6)(15) = \text{pole}(4.7)$$

$$54 = 4.7\text{pole}$$

$$\frac{54}{4.7} = \text{pole}$$

Thus the pole is approximately 11.48936 feet high.

Worksheet 46.1 Similar Polygons

The polygons in each pair are similar. Find the missing side length.

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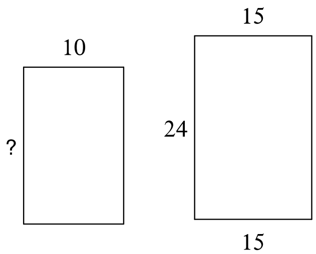

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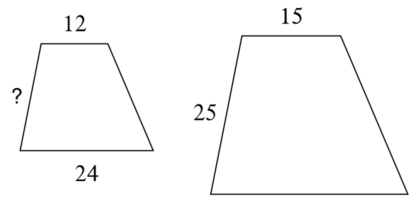

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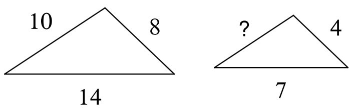

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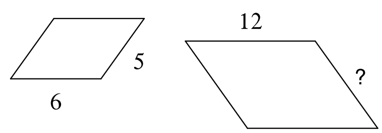

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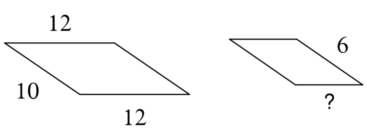

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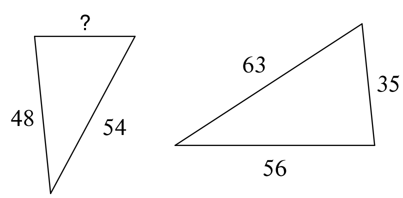

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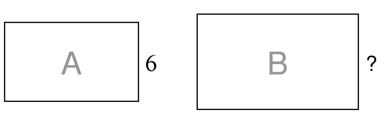
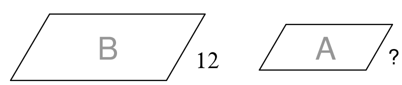


scale factor from A to B = 2 : 7

8)



scale factor from A to B = 2 : 3

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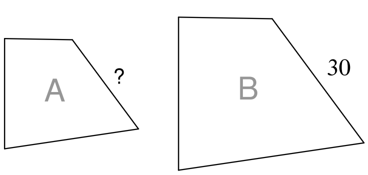
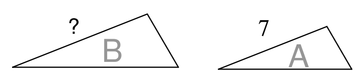


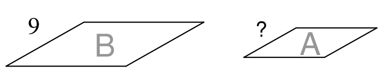
scale factor from A to B = 5 : 6

10)



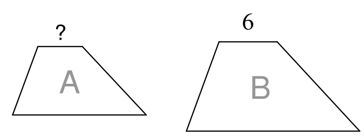
scale factor from A to B = 1 : 7

11)



scale factor from A to B = 2 : 3

12)



scale factor from A to B = 1 : 2

Worksheet 46.1 Answers

The polygons in each pair are similar. Find the missing side length.

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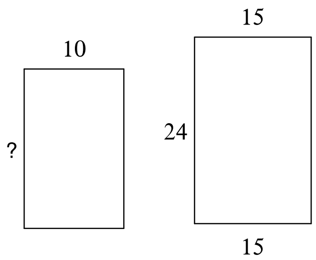

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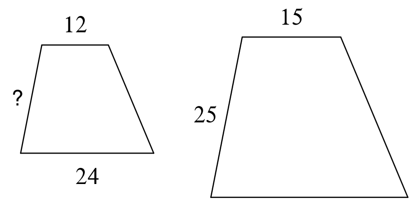
2)



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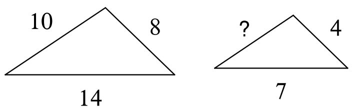
3)



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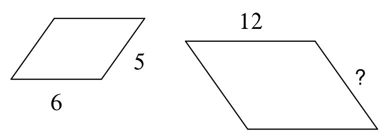

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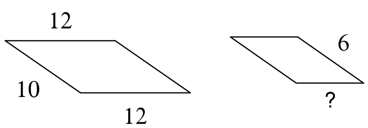
5)



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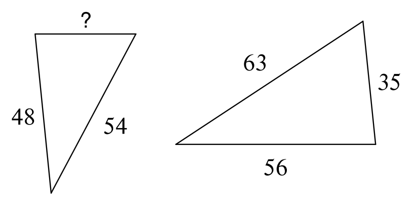

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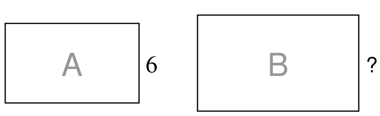
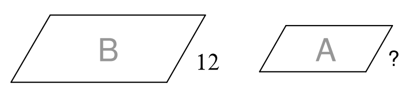


scale factor from A to B = 2 : 7

21

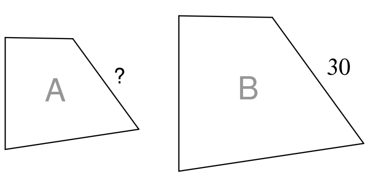
8)



scale factor from A to B = 2 : 3

8

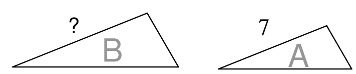
9)



scale factor from A to B = 5 : 6

25

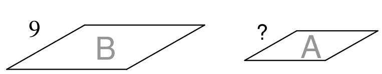
10)



scale factor from A to B = 1 : 7

49

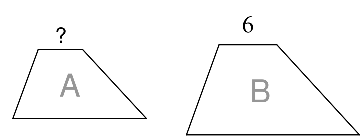
11)



scale factor from A to B = 2 : 3

6

12)



scale factor from A to B = 1 : 2

3

Exit Ticket

How long is this ladder?

1. How long is the ladder? Give your best guess.
2. Give a guess that is too high.
3. Give a guess that is too low.
4. Share your guesses with a neighbor. How do their guesses compare to yours?
5. Work with a partner to try to get a more exact estimate as to the length of the ladder. Make sure to show your reasoning and work.

Students should share their final estimates and reasoning in groups or to the class.

To see the actual measurement, check out the final solution picture at Yummymath.com.

Brought to you by **Yummymath.com**

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Application Activity

robertkaplinsky.com http://robertkaplinsky.com/work/what-should-the-freeway-sign-show/

What Should The Freeway Sign Show?

The Situation

A freeway sign is missing the distances to the upcoming exits.

The Challenge(s)

- What distances should the freeway sign display?

Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What units do freeway signs usually use to measure distances in the United States?
- What types of numbers are usually written on freeway signs?
- What fraction denominators are options to use?
- How can we determine why our distances are different than the ones on the freeway sign?
- What are possible explanations for the differences?

Consider This

This lesson is designed to get students arguing about mathematics by using a rich context. The basic premise is that students will compare the freeway sign to the corresponding map to determine the distances that should be listed. Many assumptions will need to be discussed as these non-driving students may not have enough experience to know that:

- distances are listed in miles
- distances are listed as fractions, whole numbers, or mixed numbers
- fractions are only listed as fourths, thirds, or halves

Other issues that are guaranteed to come up during the lesson include:

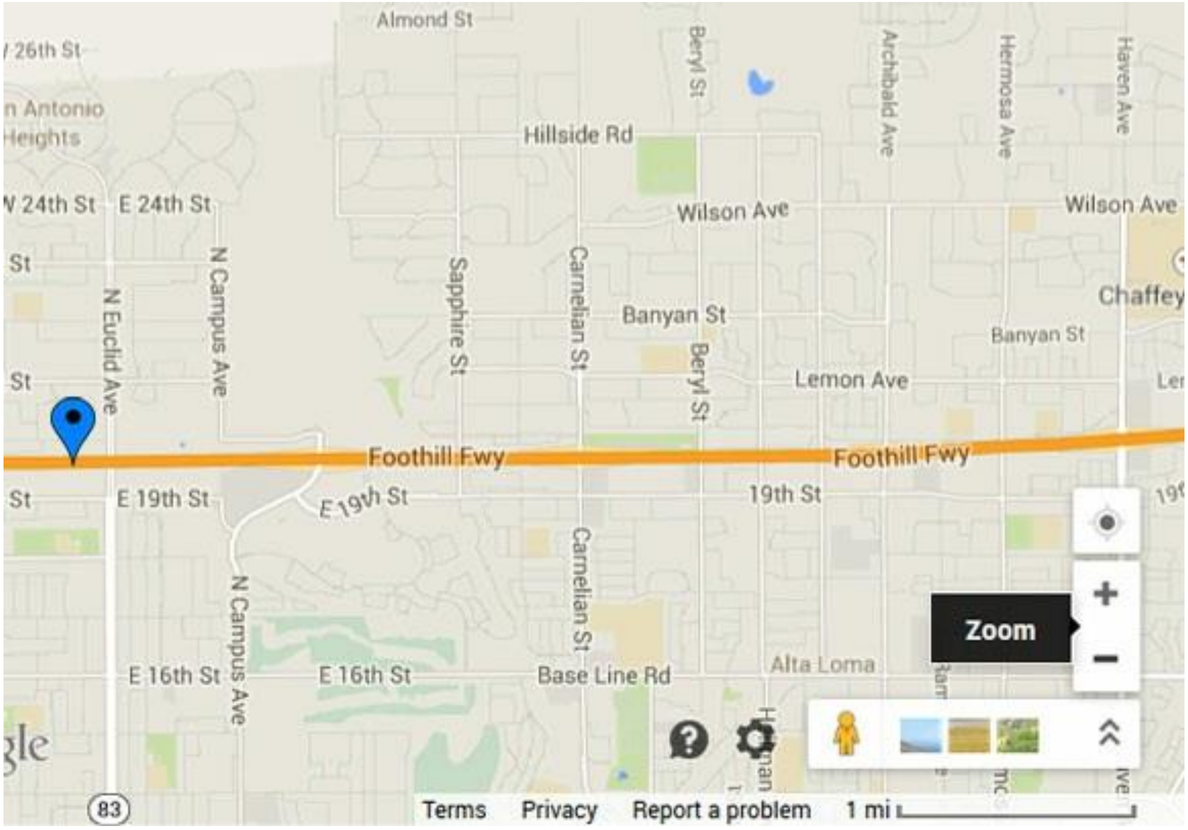
- how to correctly use a ruler
- how to choose which units to use on a ruler
- how to deal with the issues that come from measuring using decimals (cm) or mixed numbers (inches)
- how to correctly read and use a map including the scale

Also, students may find it useful to make a fraction number line that lists all the possible fractions (and their decimal equivalents) on a freeway sign including 0, 1/4, 1/3, 1/2, 2/3, 3/4, 1, 1 1/4, 1 1/3, 1 1/2, etc. so that students can have a tool to help determine which fraction is closest to the value they came up with. Look out for students who extend their number line towards the negatives. Since distance cannot be negative, the values should all be positive. Perhaps we should call it a number ray.

Map 1

This map involves a reasonably horizontal freeway with perpendicular streets running across it from north to south. Note that Campus Ave has a very peculiar path and appears to go around something as it approaches the freeway. The other two streets, Carnelian St and Archibald Ave are straighter on this map. The scale to show the length of 1 mile is on the bottom right of the map picture.


Campus Ave
Carnelian St
Archibald Ave
Foothill Fwy



Using the two images above, I chose to use centimeters and measured one mile to be 3 cm. I measured the distances from the sign as follows:

- Campus Ave – 3.1 cm

Since every 3 cm is one mile, then:

- Campus Ave – 3.1 cm / 3 cm ≈ 1.03 miles
- Carnelian St – 6.4 cm / 3 cm ≈ 2.13 miles
- Archibald Ave – 10.3 cm / 3 cm ≈ 3.43 miles

Next we need to convert those distances to mixed numbers with denominators of 2, 3, or 4:

- Campus Ave – 1.03 miles is very close to 1 mile so I chose a distance of 1
- Carnelian St – 2.13 miles is slightly closer to 2 1/4 than to 2 so I chose a distance of 2 1/4
- Archibald Ave – 3.43 miles is closer to 3 1/2 than 3 1/3 miles so I chose a distance of 3 1/2

The image below shows the original freeway sign. Interestingly, none of my answers match the sign. I would expect similar results from students. It is now a great opportunity to use Math Practice 3 and "construct [a] viable argument and critique the reasoning of others." A lively conversation should ensue. Honestly, I don't know why my distances are different. I reason that it could be a combination of:

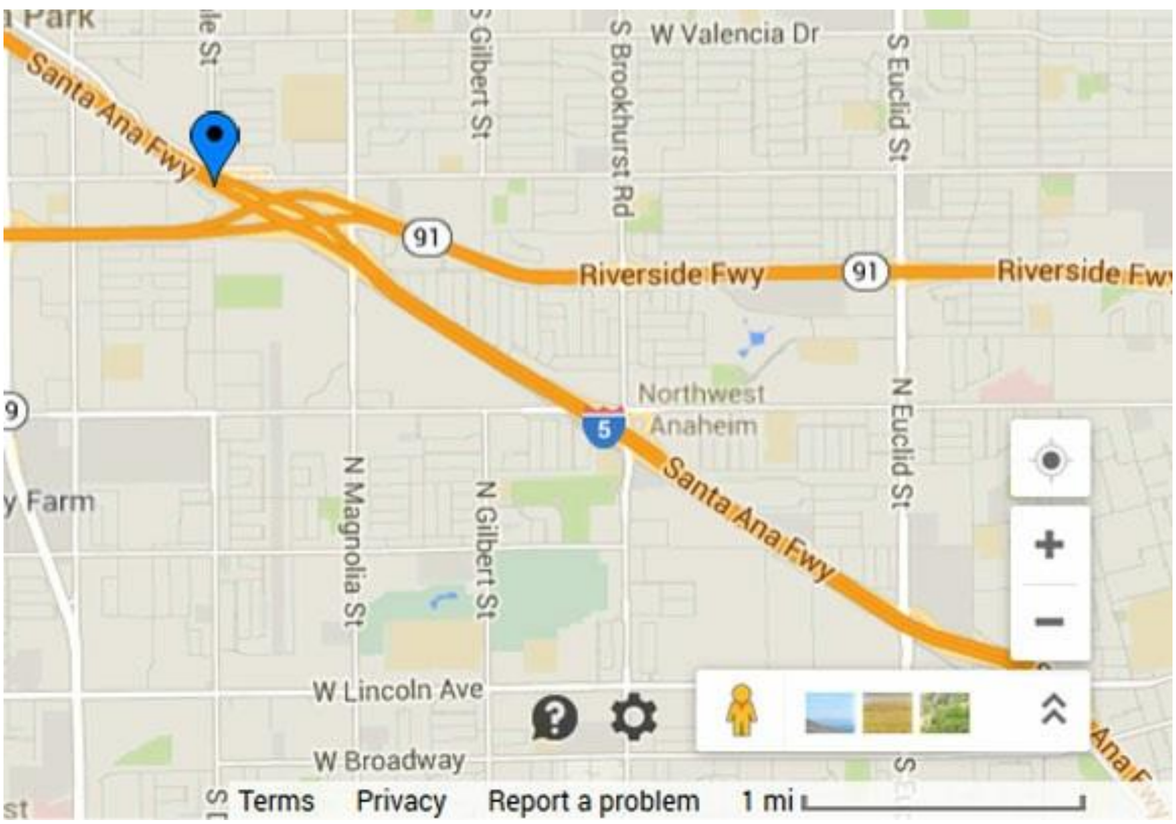
- the sign has the wrong measurements on it
- my measurements are wrong
- the sign distances were rounded up / down
- I made a calculating error

At the very least, have students defend their answers and state why the sign should be changed. Students could also debate each other if they have different distances.

Map 2

This map has a freeway running diagonally from the northwest to southeast. The other freeway (Riverside Fwy 91) that runs from east to west is not the freeway we are using. Note that there are four streets (Magnolia St, Brookhurst St, La Palma Ave, and Euclid St) and only three distances on the freeway sign. That is because Brookhurst St and La Palma Ave have the same distance listed. Every street except La Palma Ave runs from north to south. La Palma Ave is not labeled on the map but it runs from east to west and intersects Brookhusrt St perpendicularly where the “5” is located. The scale to show the length of 1 mile is on the bottom right of the map picture.


Magnolia Ave
Brookhurst St
La Palma Ave
Euclid Ave
Santa Ana Fwy


Santa Ana Fwy
S Gilbert St
S Brookhurst Rd
W Valencia Dr
S Euclid St
91
Riverside Fwy
Northwest
Anaheim
5
N Euclid St
N Magnolia St
N Gilbert St
W Lincoln Ave
W Broadway
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1 mi

Using the two images above, I chose to use centimeters and measured one mile to be 3.6 cm. I measured the distances from the sign as follows:

- Magnolia St – 2.1 cm
- Brookhurst Rd / La Palma Ave – 6.4 cm
- Euclid St – 10.1 cm

Since every 3.6 cm is one mile, then:

- Magnolia St – 3.1 cm / 3.6 cm ≈ 0.58 miles
- Brookhurst Rd / La Palma Ave – 6.4 cm / 3.6 cm ≈ 1.78 miles
- Euclid St – 10.3 cm / 3.6 cm ≈ 2.81 miles

Next we need to convert those distances to mixed numbers with denominators of 2, 3, or 4:

- Magnolia St – 0.58 (technically 0.5833333…) miles is right in the middle of 1/2 and 2/3 and I chose a distance of 1/2
- Brookhurst Rd / La Palma Ave – 1.78 miles is very close to 1 3/4 so I chose a distance of 1 3/4
- Euclid St – 3.43 miles is closer to 3 1/2 than 3 1/3 miles so I chose a distance of 3 1/2

The image below shows the original freeway sign. Again, none of my answers match the sign and I would expect similar results from students. It is still a great opportunity to use Math Practice 3 and discuss whether the answers are off because of a combination of:

- the sign has the wrong measurements on it
- my measurements are wrong
- the sign distances were rounded up / down
- I made a calculating error

At the very least, have students defend their answers and state why the sign should be changed. Students could alsc debate each other if they have different distances.

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