Weekly Focus: Pythagorean Theorem
Weekly Skill: application

LESSON 42: The Pythagorean Theorem

Lesson Summary: For the warm up, students will solve a problem about distance. In Activity 1, they will review the classification of triangles. In Activity 2, they will learn the Pythagorean Theorem. In Activity 3, they will solve word problems with the Pythagorean Theorem. Activity 4 is an application activity related to hiking and the steepness of inclines. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 42:

- Video (length 8:48) on the Pythagorean Theorem. The video is required for teachers and recommended for students.
- Notes on Classifying Triangles
- 2 Worksheets (42.1, 42.2) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 96– 97)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 130 – 133)
- Application Activity on measuring the steepness of hikes. Note: Please download the application activity directly from yummy math: https://www.yummymath.com/2016/steepness-and-fall-hiking/

Objectives: Students will be able to:

- Solve the distance word problem
- Practice classifying triangles by names
- Learn and practice the Pythagorean Theorem with computation and word problems
- Do a real-life application of the Pythagorean Theorem

ACES Skills Addressed: N, CT, LS, ALS
CCRS Mathematical Practices Addressed: Building Solution Pathways, Mathematical Fluency, Use Appropriate Tools Strategically
Levels of Knowing Math Addressed: Intuitive, Abstract, Pictorial and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 42 Warm-up: Solve the distance problem	**Time: 5-10 Minutes**

Write on the board: A school measures 80 feet long and 52 feet wide.

Basic Question:

- How many laps must a runner run around the school to run a mile?
- Notes:
    - Give students a hint that 1 mile = 5,280 feet if they need it, or have them Google it.
    - Answer: Perimeter is 264 feet so 5280/264 = 20 laps
    - Have volunteers write how they solved the problem on the board. Some may have used proportions.

Extension Questions:

- Write an equation for the problem.
    - 5,280 ÷ [2 (80) + 2 (52)] = 20 laps. Answers may vary.
- Running 20 laps around the school would equal how many yards?
    - 1760 yards (Since 20 laps = 1 mile, we just need to know that 1 mile has 1,760 yards)

Lesson 42 Activity 1: Classify Triangles Review	**Time: 15 Minutes**

1. Students were introduced to the different types of triangles in the last lesson.
2. Review the classification of triangles with the attached **Notes on Classifying Triangles**. You can explain and have students take notes.
3. Do **Worksheet 42.1**. Do the first example together and then students can work individually.

Lesson 42 Activity 2: Introduction to Pythagorean Theorem	**Time: 10 Minutes**

1. Write this example on the board (a variation from the video): The base of a painter's ladder is 21 feet from the house. When leaned against the side of the house, it reaches a height of 28 feet. How tall is the ladder?

2. Draw a house with a ladder leaning on it. The ladder is the side opposite the right angle. It is called the **hypotenuse**. The other two sides are called the **legs.** One of those sides is the side of the house and the other is the distance on the ground from the house to the base of the ladder.
3. The **Pythagorean Theorem** helps solve this problem. It states that in a right triangle, the square of the hypotenuse is equal to the square of each leg added together. The legs are a and b and the hypotenuse is c. The equation is $\mathbf{a^2 + b^2 = c^2}$.
4. In this example:
   a. $21^2 + 28^2 = x^2$
   b. $441 + 784 = x^2$
   c. $1225 = x^2$
   d. $x = 35$ feet. The ladder is 35 feet tall.

5. Now let's try finding the hypotenuse using the same situation. Write the following example on the board (it is the same as the video): A painter is at the top of a 35-foot ladder painting a house. The base of the ladder is 21 feet from the house. How far down would he fall if he fell off the ladder?
6. In this example:
   a. $21^2 + x^2 = 35^2$
   b. $441 + x^2 = 1225$
   c. $x^2 = 1225-441$
   d. $x^2 = 784$
   e. $x = 28$ feet. The painter would fall 28 feet down.
7. Students already have notes on the Pythagorean Theorem from the last lesson.
8. Do **Worksheet 42.2.** All of the problems are to solve for the hypotenuse.

Lesson 42 Activity 3: Solve Word Problems	**Time: 60 Minutes**

1. Do the problems in the **student book pages 96-97** together. (15 minutes)
2. Students can do the problems in the **workbook pages 130-133**. Note that questions 3 to 6 connect geometry to graphing and algebra. (35 minutes)
3. Solve some of the more challenging problems on the board together. (10 minutes)

Lesson 42 Activity 4: Application: Steep Hikes	**Time: 20-30 Minutes**

4. This activity can be downloaded directly from the site yummy math as indicated on the first page of the lesson. A copy is attached here for reference.
5. Introduce the activity by asking students if they have ever seen signs on highways showing steep inclines. Did they understand what those meant?

6. This real-life application activity will answer that question as well as review slope, mean (average), percent, and make use of the Pythagorean Theorem.
7. Question 11 could be assigned as extra homework.

Notes on Classifying Triangles

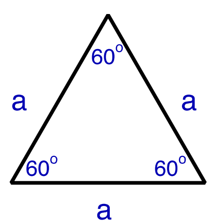
Triangles

- **A triangle has three sides and three angles.**
- **The three angles always add up to 180 degrees.**

Equilateral, Isosceles and Scalene

There are three special names given to triangles that tell how many sides and angles are equal.

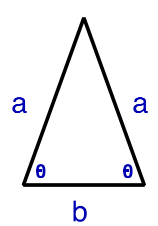
There can be 3, 2, or NO equal sides and angles:



Equilateral Triangle

Three equal sides. a

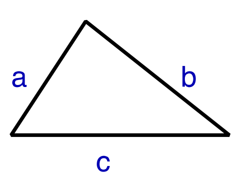
Three equal angles, always 60^{o}.



Isosceles Triangle

Two equal sides. a

Two equal angles. θ



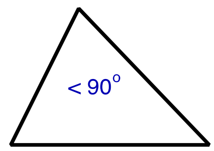
Scalene Triangle

No equal sides.

No equal angles.

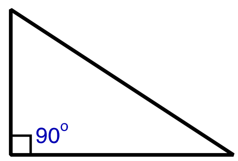
What Type of Angle ?

Triangles can also have names that tell you what type of angle is inside:



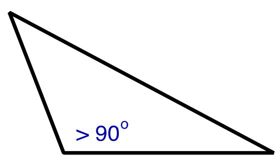
Acute Triangle

All angles are less than 90^o.



Right Triangle

Has a right angle (90^o)



Obtuse Triangle

Has an angle greater than 90^o.

Combining the Names.

Sometimes a triangle may have two names, here is a list of possible combinations:

Right Scalene Triangle

Has a right angle (90^o), and no equal sides or angles.

Right Isosceles Triangle

Has a right angle (90^o) and two equal angles (45^o), and two equal sides.

Obtuse Scalene Triangle

Has an angle $> 90^o$, and no equal sides or angles.

Obtuse Isosceles Triangle

Has an angle $> 90^o$, and two equal sides.

Acute Scalene Triangle

Has all angles $< 90^o$, and no equal sides or angles.

Acute Isosceles Triangle

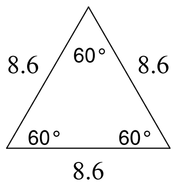
Has all angles $< 90^o$, and two equal sides.

Math-Aids.Com

Worksheet 41.1 Classify Triangles

Classify each triangle by each angles and sides.

7)



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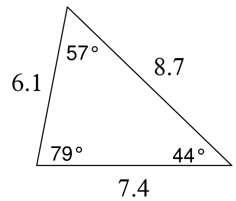

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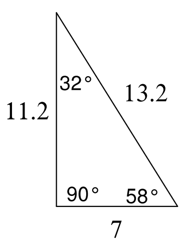

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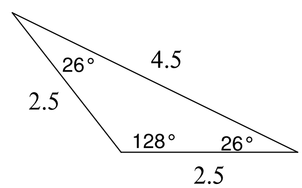

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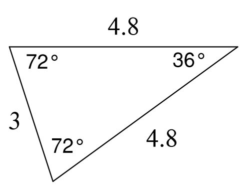

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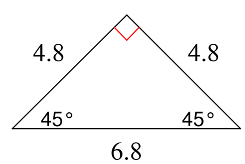
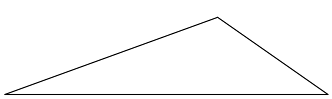


Classify each triangle by each angles and sides. Equal sides and equal angles, if any, are indicated in each diagram.

13)



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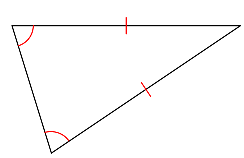
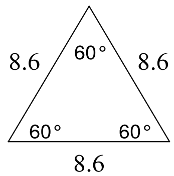


Worksheet 41.1 Answers

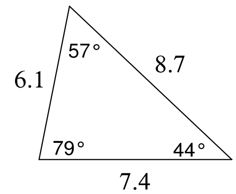
Classify each triangle by each angles and sides.

7)



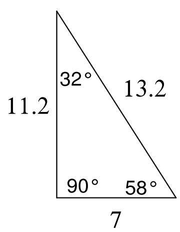
equilateral

8)



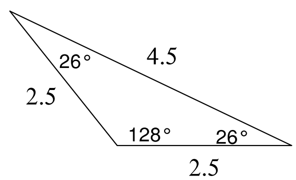
acute scalene

9)



right scalene

10)



obtuse isosceles

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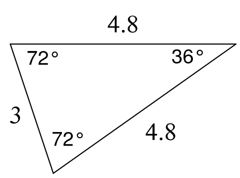
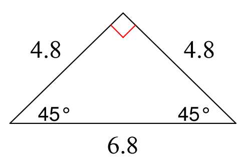


acute isosceles

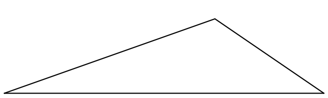
12)



right isosceles

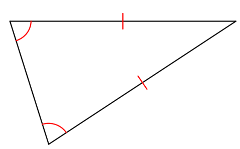
Classify each triangle by each angles and sides. Equal sides and equal angles, if any, are indicated in each diagram.

13)



obtuse scalene

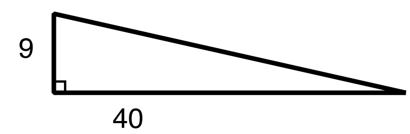
14)

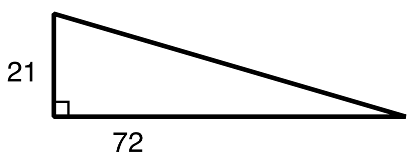


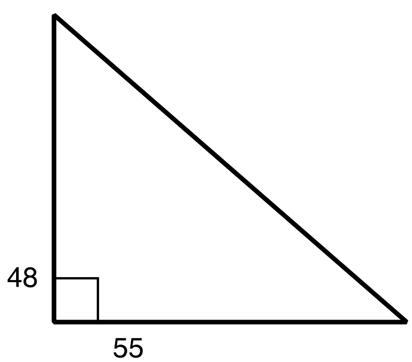
acute isosceles

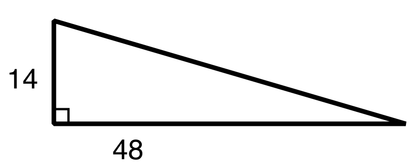
Worksheet 42.2 Find the Hypotenuse

Find the length of the third side of each triangle.

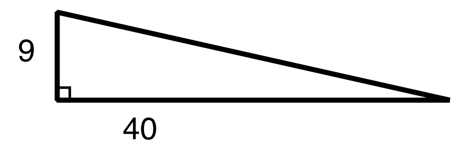








Worksheet 42.2 Answers



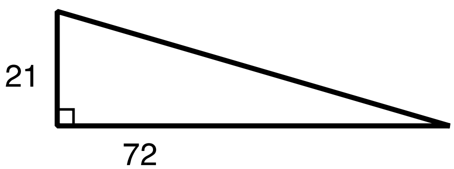
$9^2 + 40^2 = c^2$

$81 + 1600 = c^2$

$1681 = c^2$

$\sqrt{1681} = c$

$41 = c$



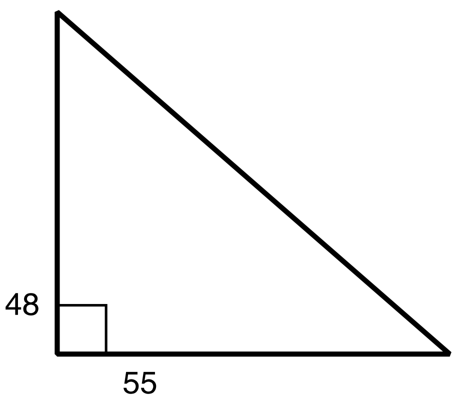
$21^2 + 72^2 = c^2$

$441 + 5184 = c^2$

$5625 = c^2$

$\sqrt{5625} = c$

$75 = c$



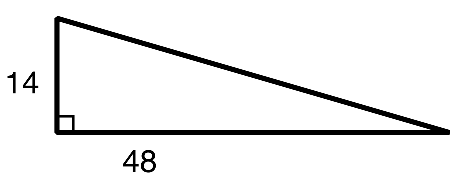
$48^2 + 55^2 = c^2$

$2304 + 3025 = c^2$

$5329 = c^2$

$\sqrt{5329} = c$

$73 = c$



$14^2 + 48^2 = c^2$

$196 + 2304 = c^2$

$2500 = c^2$

$\sqrt{2500} = c$

$50 = c$

Application: Steep Hikes

Steep hikes

Fall is such a nice time of year for hiking. The mosquitoes are usually absent because of early frosts at the higher elevations, the tree colors are beautiful, and the temperatures are cool.

This year when I went hiking I encountered some weird numbers that I didn't know how to interpret. I did some research and am excited to share it with you.

During the drive to the mountains, when the road had a steep section, I saw warning signs like this one.

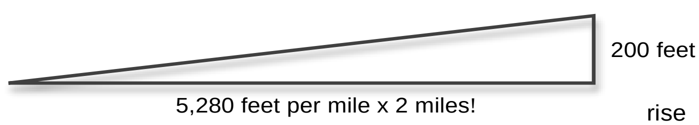
What does that mean? 6% doesn't sound very steep but it was on a warning sign so maybe that is very steep.

Then at the hike trailhead there was another mention of percent grade change. Are these the same things? What percent change is steep and what is just moderate?

I looked up percent grade change and found two different ways of calculating it. Bicyclists, road builders, and hikers all use this notion.

During road construction, surveying equipment is used to find the change in vertical climb of the road as compared to the horizontal distance of the road. The steepness of a road is just like the slope of a line only it is usually expressed in percent. The steepness of a road has a lot to do with its safety. Bicyclists will need to use breaks constantly on a steep incline. Heavy trucks will labor up and speed down. So, like the slope of a line, the percent elevation change is $\frac{\text{rise}}{\text{run}}$ and it significantly effects the safety of driving on a road. Slope is a fraction but percent grade change is slope x 100% so that It becomes a percent.

If a road changes elevation by 200 feet in 2 miles then it's slope is;

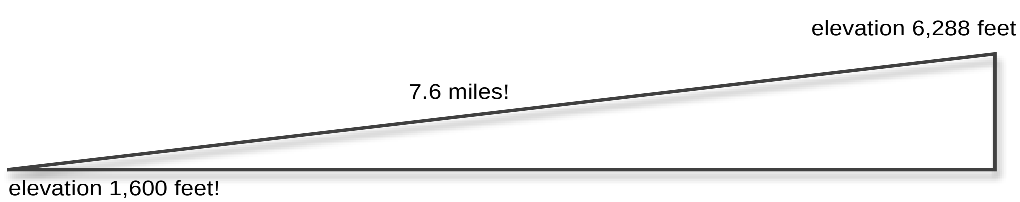


$$\frac{\text{rise}}{\text{run}} = \frac{200 \text{ feet}}{2 \text{ miles} \cdot 5280 \text{ feet per mile}} = .0189 \approx 1.9\% \text{ grade change}$$

1. We drove through Franconia Notch, NH to get to the mountain we were going to hike, Mt Pierce. The roadway rose in elevation from 1,000 feet to 1,950 feet in about 4 miles. What is the Parkway's average percent grade change? Please show your work.

The Mount Washington Auto Road is famous for ruining the transmission of automobiles or burning out their brake pads on the way down. The roadway begins on Route 16 in Glen, NH at 1,600 feet elevation and rises to a parking area just below the summit of Mt Washington at 6,288 feet in 7.6 miles of curvy steep roadway.

2. What is the percent grade change of this roadway?



Oh no, I don't have the horizontal measure from the base of the mountain to right under the peak. But, I think I can use the Pythagorean Theorem to figure out the run. Remember: $a^2 + b^2 = c^2$?

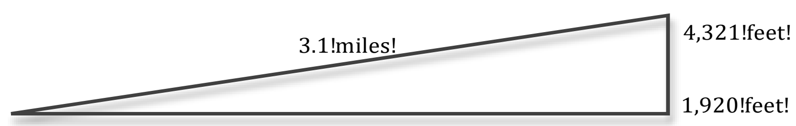
3. Use your calculator and the Pythagorean Theorem to find what must be the distance from the base of the Mount Washington Auto Road to directly under the peak of Mount Washington.

4. Now use the calculated rise and run to find the average slope of the road.

5. Change the slope into percent grade change.

This same situation occurs when calculating the percent grade change of a hike. I guess it is not very easy to measure the horizontal distance from the base of the trail to directly under the peak of the mountain.

Our climb up Mt Pierce was 3.1 miles long. That sounds a lot easier than it was. The elevation gain was from 1,920 feet to 4,312 feet. The measure 3.1 miles was not the horizontal change from where we started to right under the peak. 3.1 miles was the actual trail length … the hypotenuse of the right triangle shown below.



6. Use your calculator and the Pythagorean Theorem again to figure the base of the right triangle shown above. That is the run of this calculation.

7. What is your calculated rise?

8. What is the slope of this hike?

9. What is the percent grade change?

10. Now that you've calculated percent grade change for 3 situations, make some conclusions about the percent grade change of a steep road, a moderate trail, and an easy bike path.

11. Do some research and find examples of percent grade changes for two of the above situations. Show or explain how you determined your answer.

Source: White Mountain Guide

Brought to you by Yummymath.com

Application Activity Answers

Steep hikes

Fall is such a nice time of year for hiking. The mosquitoes are usually absent because of early frosts at the higher elevations, the tree colors are beautiful, and the temperatures are cool.

This year when I went hiking I encountered some weird numbers that I didn't know how to interpret. I did some research and am excited to share it with you.

During the drive to the mountains, when the road had a steep section, I saw warning signs like this one.

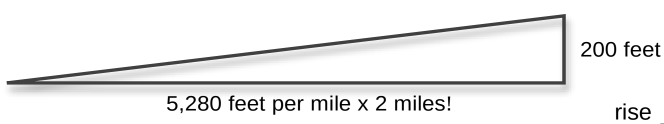
What does that mean? 6% doesn't sound very steep but it was on a warning sign so maybe that is very steep. 6% is actually the maximum percent grade change for the US for highway system.

Then at the hike trailhead there was another mention of percent grade change. Are these the same things? What percent change is steep and what is just moderate?

I looked up percent grade change and found two different ways of calculating it. Bicyclists, road builders, and hikers all use this notion.

> During road construction, surveying equipment is used to find the change in vertical climb of the road as compared to the horizontal distance of the road. The steepness of a road is just like the slope of a line only it is usually expressed in percent. The steepness of a road has a lot to do with its safety. Bicyclists will need to use breaks constantly on a steep incline. Heavy trucks will labor up and speed down. So, like the slope of a line, the percent elevation change is $\frac{\text{rise}}{\text{run}}$ and it significantly effects the safety of driving on a road. Slope is a fraction but percent grade change is slope x 100% so that It becomes a percent.

If a road changes elevation by 200 feet in 2 miles then it's slope is;



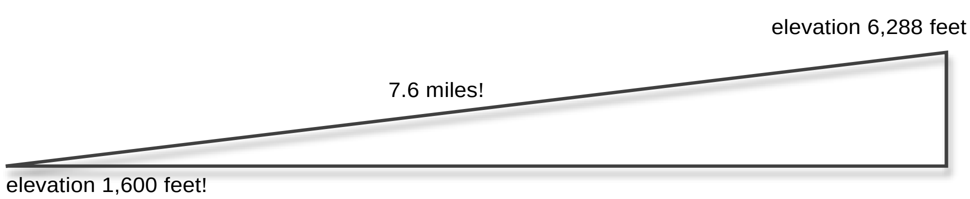
$$\frac{\text{rise}}{\text{run}} = \frac{200 \text{ feet}}{2 \text{ miles} \cdot 5280 \text{ feet per mile}} = .0189 \approx 1.9\% \text{ grade change}$$

1. We drove through Franconia Notch, NH to get to the mountain we were going to hike, Mt Pierce. The roadway rose in elevation from 1,000 feet to 1,950 feet in about 4 miles. What is the Parkway's average percent grade change? Please show your work.

$$\frac{1{,}950 - 1{,}000}{4 \cdot 5280} = \frac{950}{21{,}120} = .04498 \approx 4.5\% \text{ grade change}$$

The Mount Washington Auto Road is famous for ruining the transmission of automobiles or burning out their brake pads on the way down. The roadway begins on Route 16 in Glen, NH at 1,600 feet elevation and rises to a parking area just below the summit of Mt Washington at 6,288 feet in 7.6 miles of curvy steep roadway.

2. What is the percent grade change of this roadway?



Oh no, I don't have the horizontal measure from the base of the mountain to right under the peak. But, I think I can use the Pythagorean Theorem to figure out the run. Remember: $a^2 + b^2 = c^2$?

3. Use your calculator and the Pythagorean Theorem to find what must be the distance from the base of the Mount Washington Auto Road to directly under the peak of Mount Washington.

$$a^2 + b^2 = c^2$$
$$a^2 + (6{,}288 - 1600)^2 = (7.6 \cdot 5{,}280)^2$$
$$a^2 + 21{,}977{,}344 = 1{,}610{,}256{,}384$$
$$a^2 = 1{,}588{,}279{,}040$$
$$a = 39{,}853.2 \text{ feet}$$

4. Now use the calculated rise and run to find the average slope of the road.

$$m = \frac{6{,}288 - 1600}{39{,}853} = \frac{4688}{39{,}853} = .1176$$

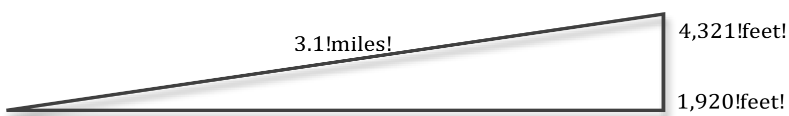
5. Change the slope into percent grade change.

$$.1176 = 11.8\%$$

This same situation occurs when calculating the percent grade change of a hike. I guess it is not very easy to measure the horizontal distance from the base of the trail to directly under the peak of the mountain.

Our climb up Mt Pierce was 3.1 miles long. That sounds a lot easier than it was. The elevation gain was from 1,920 feet to 4,312 feet. The measure 3.1 miles was not the horizontal change from where we started to right under the peak. 3.1 miles was the actual trail length … the hypotenuse of the right triangle shown below.

!



!

6. Use your calculator and the Pythagorean Theorem again to figure the base of the right triangle shown above. That is the run of this calculation.

vertical change = 4,321-1,920 = 2,401
3.1 miles = 3.1 x 5,280 feet = 16,368

$$(2{,}401)^2 + a^2 = (16{,}368)^2$$
$$5{,}764{,}801 + a^2 = 267{,}911{,}424$$
$$a^2 = 262{,}146{,}623$$
$$a = 16{,}190.94$$
$$a \approx 16{,}191$$

7. What is your calculated rise?

vertical change = 4,321-1,920 = 2,401

8. What is the slope of this hike?

$$m = \frac{2{,}401 \text{ feet}}{16{,}191 \text{ feet}} = .1482$$

9. What is the percent grade change?

.1482!=!14.8!%

10. Now that you've calculated percent grade change for 3 situations, make some conclusions about the percent grade change of a steep road, a moderate trail, and an easy bike path.

Of course, answers will vary. These are my answers;

- Franconia Notch is a steep road and it's percent grade change is 4.5%. I've heard that highways can only be 6% grade change for safety reasons. So, 4.5% must be pretty steep.

- The hiking trail to the top of Mt Pierce was 14% and Ms. Lewis isn't that vigorous a person. So a trail of 14% grade must be steep but not awful.
- Bicyclists probably prefer an incline of 4 or 5 %. I like riding on flat or only moderately rising slopes.

11. Do some research and find some examples of percent grade changes for one or two of the above situations. Show or explain how you determined your answer.

There will be lots of interesting answers and quotes. Here are a few that I found.

Damnation Creek Trail

- Location: Del Norte Coast Redwoods State Park, CA
- Trailhead: Milepost 16.0 on Hwy 101
- Mileage: 2.2
- Difficulty Level: Strenuous, numerous steep grades and switchbacks. Trail drops 1,000 feet (16 percent grade)

Description: Experience the ancient redwood forest and the jagged Pacific coastline. This steep trail descends 1,000 feet (330 m) through the forest where canopy branches look like treetop arms holding thousand of plants. In the past, Tolowa Indians used the tidepools at the ocean for food gathering. Arrive at low tide and carefully make your way to the beach from the bluff. Remember our motto for tidepool creatures, observe but do not disturb.

Trail construction from the Federal Highway commission;

- If the steepest grade on the trail cannot be less than 20 percent, the segment should be as short as possible and the remainder of the trail should comply with the recommendations;
- If there is a segment of trail that has a 10 percent grade for more than 9.14 m (30 ft), a level rest interval should be provided as soon as possible, and the remainder of the trail should be designed according to the recommendations;
- If there is a segment of trail that has a cross slope of more than 5 percent, the segment should be as short as possible and the remainder of the trail should follow the recommended specifications; or

Source: White Mountain Guide

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