

## Lesson 39: Comparison of Functions

### LESSON 39: Comparison of Functions

**Weekly Focus:** functions  
**Weekly Skill:** comparison and application

**Lesson Summary:** For the warm-up, students will solve a problem about hiking. Activity 1 is to help students compare functions that are in table vs. graph format. Activity 2 includes functions in various forms with related questions. Activity 3 consists of examples of what happens when graphs are manipulated in different ways. Activity 4 provides more challenging practice problems in the student book and workbook. Activity 5 is an application of functions in which students compare different cell phone plans. Estimated time for the lesson is 2 hours.

#### Materials Needed for Lesson 39:

- Video (length 5:15) on comparison of functions. The video is required for teachers, optional for students.
- Notes 39A sheet (attached)
- 3 Worksheets (39.1, 39.2, and 39.3) with answers (attached)
  - Worksheet 39.3 is to be used for examples for Activity 3.
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Student Book (pages 82 – 83)*
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook (pages 122 – 125)*
- Application Activity (links embedded in lesson plan)
- Graph paper for the application activity
- Notes:
  - The application activity comes from the website [yummymath.com](http://www.yummymath.com). They request that each teacher/coordinator download the lesson each time for their own use:  
<http://www.yummymath.com/wp-content/uploads/Cell-plans-i6.pdf>
  - A copy of the lesson and answers is provided here for reference.
  - Advise students that this is the last new lesson in the algebra unit. The next lesson will be an algebra test.

**Objectives:** Students will be able to:

- Solve the word problem about hiking, time, and distance as well as graph it
- Compare functions represented as tables, graphs, equations, and/or verbal statements
- Apply their skills by making tables and graphs of different cell phone plans

**ACES Skills Addressed:** N, CT, LS, EC

**CCRS Mathematical Practices Addressed:** Building Solution Pathways, Mathematical Fluency, Reason Abstractly and Quantitatively, Look For and Make Use of Structure

**Levels of Knowing Math Addressed:** Intuitive, Pictorial, Abstract, and Application

#### Notes:

You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The “easier” problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

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Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

### Lesson 39 Warm-up: Solve the hiking problem

**Time: 5 Minutes**

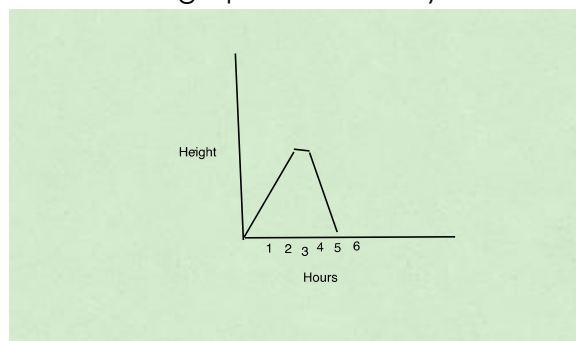
Write on the board: Courtney hikes up a mountain peak at a speed of 2.5 miles per hour and it takes her 3 hours to reach the top. She rests for a half hour and takes 1.75 hours to go back down the same route.

#### Basic Questions:

- How many miles has she hiked total? (*2.5 mi. x 3 hours = 7.5 mi. each way = 15 miles total*)
- If she starts at 8:00 a.m., at what time is done hiking? (*3 hours up + 30 min. rest + 1 hr. 45 min. down = 5 hr. 15 min. of hiking so she is done at 1:15 pm*)

#### Extension Question:

- Draw a line graph of Courtney's hike. *Students' graphs should look similar to:*



### Lesson 39 Activity 1: Comparing Functions in Different Formats

**Time: 20 Minutes**

- Review the meaning of functions and their different representations:
  - Each input  $x$  has exactly one output  $f(x)$ .
  - Functions can be represented with tables, equations, graphs and/or verbal statements.

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- c. They can be compared via their slopes, intercepts, minimums and maximums.
- d. You can compare 2 linear functions, 2 quadratic functions, or one of each.
2. Use the attached **Notes 39A Guided Lesson** for examples and explanations.
3. Note for teacher:
  - a. The notes may be copied for students or projected on the wall.
  - b. It may be helpful to graph each of the functions listed in table format. This will help deepen students' understanding as well as practice more graphing.
4. Do **Worksheet 39.1** for practice.

**Lesson 39 Activity 2: Comparing Functions by Graphing**

**Time: 10 Minutes**

1. In this activity, students will practice graphing functions that are represented verbally or answering questions about a graph.
2. Do **Worksheet 39.2**.

**Lesson 39 Activity 3: Manipulating the Graphs of Functions**

**Time: 20 Minutes**

1. Students have experience with looking at functions and their representations from previous lessons and from Activities 1 and 2.
2. Copy **Worksheet 39.3** for the students and use the questions as examples to practice what happens when a function is manipulated.
3. Note: The worksheet has "independent practice" in the title but you won't use it as such.
4. Do the 9 questions together as necessary. If there is time, have students graph the functions to help make it easier to understand the functions.

**Lesson 39 Activity 4: Comparison of Functions Practice**

**Time: 45 Minutes**

1. Review the meanings of slope and intercepts and the different ways to find them as explained in the left column on page 82 in the student book.
2. Do **pages 82 – 83** in the **student book**.
3. Do the **workbook pages 122 – 125** for additional practice.
4. Note: These problems may be challenging for students and you may need to do many of them together as a class. If you do, still give students some time to work alone to try to figure

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them out on their own.

**Lesson 39 Application: Comparing iPhone Plans**

**Time: 20-30 Minutes**

1. The application activity can serve as a culminating activity of graphing and comparing functions. For student copies, print the activity from [this link](#).
2. Students can do it for homework if there isn't enough time in class.
3. It can, however, foster a rich discussion with critical thinking if done in class.

Get an updated Comparing iPhone Plans (iPhone 7) activity [here](#). (Answers not included.)

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Notes 39A

**Comparing Functions in Different Formats – Guided Lesson**

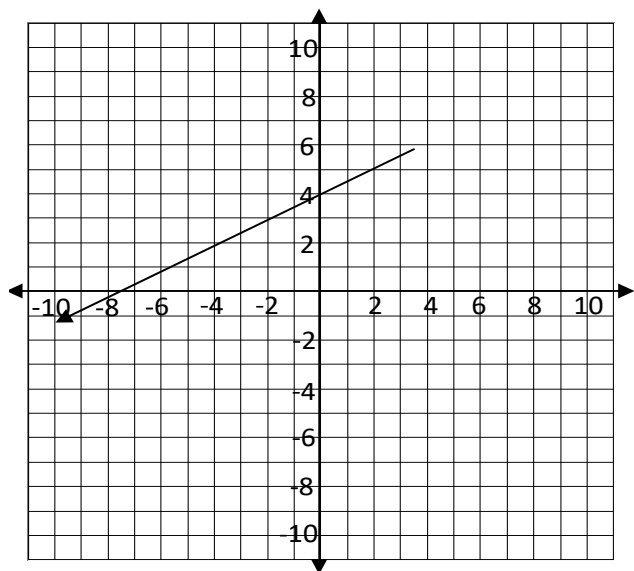
Complete the following problems:

- 1) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y-values of a quadratic function.

x	y
0	0
2	3
4	15
6	35
8	63
10	99
12	143

Function 2: The graph of a linear function is shown.



From the two functions, which function grows faster for large positive values of x?

### Comparing Functions in Different Formats – Guided Lesson Explanation

For all of these problems we are looking to compare the growth or lack of growth of the data. Growth of data can be gauged by the slope of the line created by the data. In a sense, slope indicates the rate of change.

The steeper the slope of the line created by the data, the more growth. If you are looking for lack of growth in the data, you are looking for shallow rise in the slope, when compared to another set of data. If the slope is negative, there is no growth.

Slope intercept form is defined as  $y = mx + b$

$m$  = slope                       $b$  = y-intercept (where the line crosses the y-axis)

You can also determine slope using the following equation when  $x_1$  is not equal to  $x_2$ .

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

#### Explanation #1

Step 1) Find two points where  $x_1$  is not equal to  $x_2$  for both functions.

Function 1: I chose points (2, 3) and (6, 35)

Function 2: I chose points (-2, 3) and (2, 5)

Step 2) Determine the slope ( $m$ ) for both functions based off of the points that were chosen.

$$\text{Function 1: } \frac{35 - 3}{6 - 2} = \frac{32}{4} = 8$$

$$\text{Function 2: } \frac{5 - 3}{2 - -2} = \frac{2}{4} = 0.5$$

Step 3) Interpret the difference in slope between the functions.

Function 1's slope is much greater. This indicates that Function 1 grows faster for large positive values of  $x$ .

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Worksheet 39.1 Practice Comparing Functions in Different Formats

Comparing Functions in Different Formats – Independent Practice Worksheet

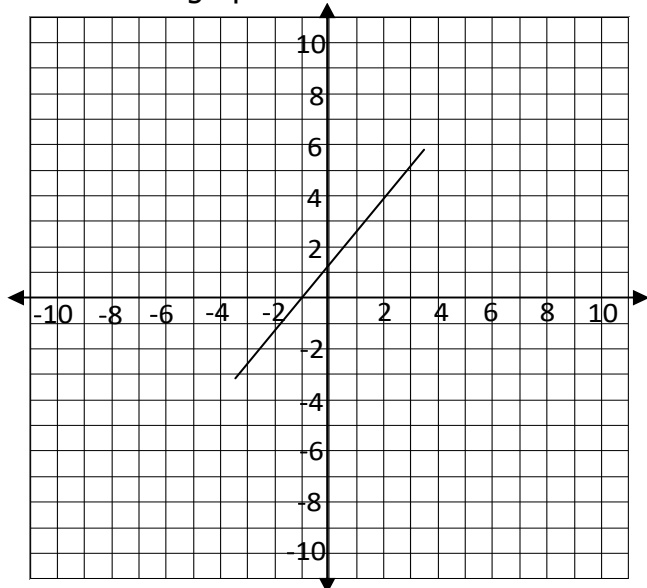
Complete all the problems.

1) Two functions are represented in different ways.

Function 1: The input-output table shows the  $x$ - and  $y$ - values of a quadratic function.

$x$	$y$
1	1
3	9
4	16
5	25
6	36
7	49
8	64

Function 2: The graph of a linear function is shown.



From the two functions, which function grows faster for large positive values of  $x$ ?

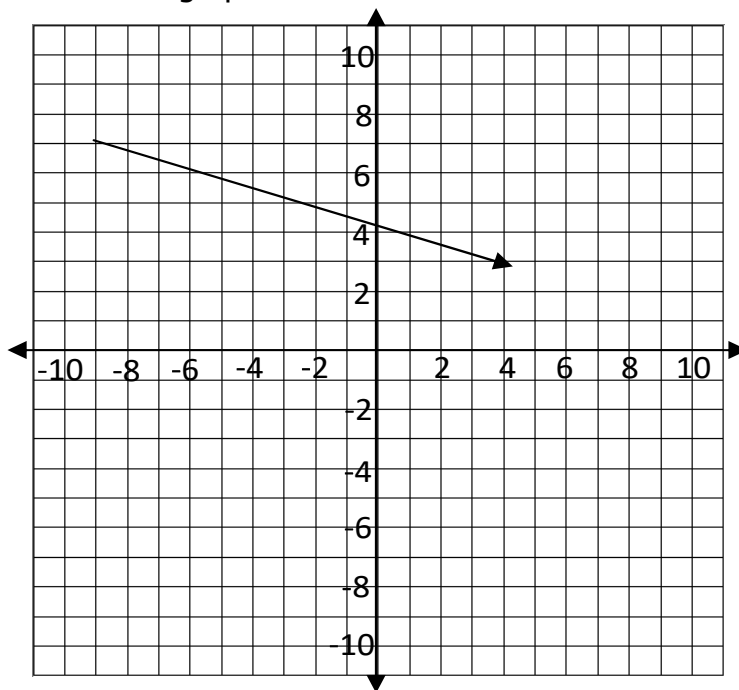
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2) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y- values of a quadratic function.

x	y
0	0
1	1
2	4
4	16
6	36
7	49
10	100

Function 2: The graph of a linear function is shown.



Which of the two functions shows no growth at all?



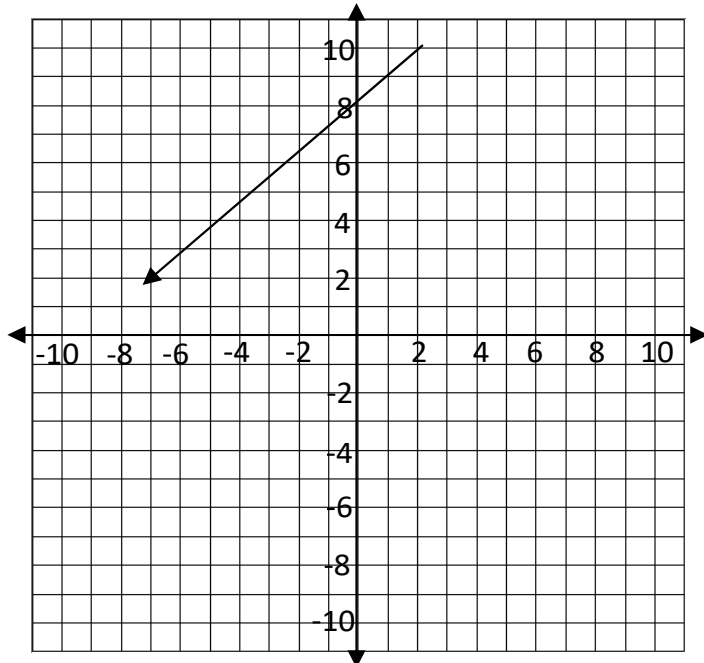
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3) Two functions are represented in different ways.

Function 1: The input-output table shows the x- and y- values of a quadratic function.

x	y
2	7
4	19
6	39
8	67
10	103
12	147
13	172

Function 2: The graph of a linear function is shown.



Which of the two functions grows at a faster rate?

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### Worksheet 39.1 **Answers**

- (1) Function 1
- (2) Function 2
- (3) Function 1

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Worksheet 39.2 Compare Functions by Graphing

**Analyzing Functional Relationships by Graphing - Independent Practice**

Complete all the problems.

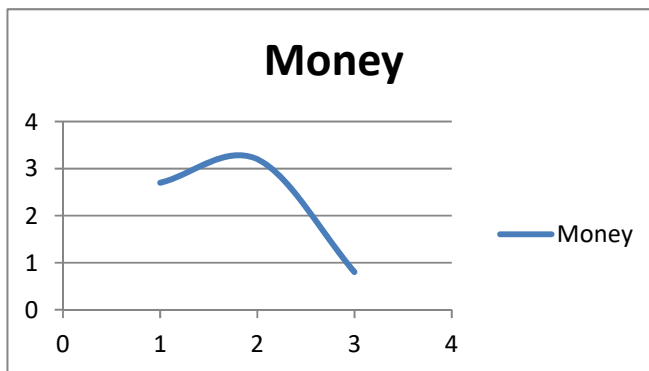
1. Michael is going to the playground. First, he walks to church. He stays there for few minutes. Then he hires a taxi to drive him to the playground. The taxi stops at a coffee shop and then the playground. Draw the graph representing Michael's trip to the playground.

2. Describe the graph of the function. Is this linear or non-linear?

x	1	2	3	4	5
y	5	10	15	20	25

3. Describe the graph of the function between  $y = 2$  and  $y = 4$ .

4. Describe the graph of the function. Is this relationship non-linear?



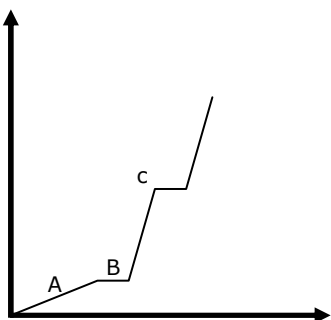
5. Isabella is a plumber. She walks to the market. She buys some tools and stays there a few minutes. Then she hires a taxi to go to Sarah's house. After Sarah's she takes the taxi to Olivia's house. Draw the graph representing Isabella's trip to Olivia's house.

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Worksheet 39.2 Compare Functions by Graphing **Answers**

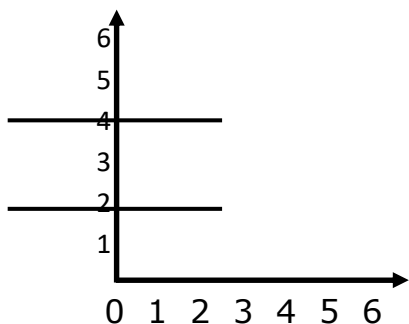
**Analyzing Functional Relationships by Graphing - Independent Practice Worksheet Answer Key**

1. Michael's trip to the playground.



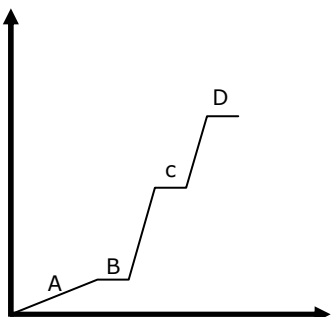
2. Linear

3.



4. Non-Linear

5. Isabella's trip to Olivia's house.



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Worksheet 39.3 Manipulating the Graphs of Functions

**Manipulating the Graphs of Functions - Independent Practice Worksheet**

Complete all the problems.

1. The function  $y = 11x - 8$ , which statement best describes the effect of increasing the y-intercept by 7?

- a. The new line is parallel to the original.
- b. The new line has greater rate of change.

2. Which statement best describes the effect on the graph of  $f(x) = 6x - 5$

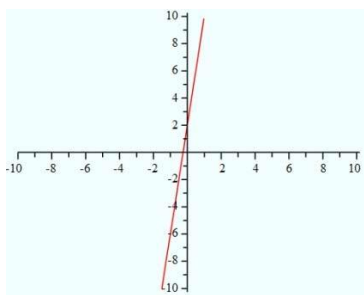
If the y intercept is changed to + 12?

- a. The slope does not change.
- b. The new line passes through the origin.

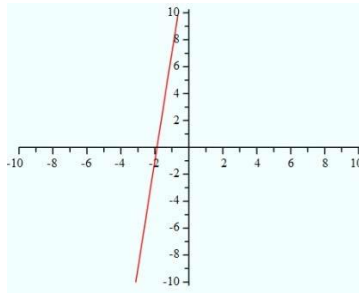
3. The graph of a line that contains the points  $(-7, -5)$  and  $(2, 4)$  is shown below.

Which best represents this line if the slope is doubled and the y-intercept remains constant?

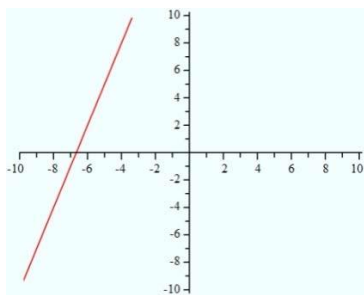
a)



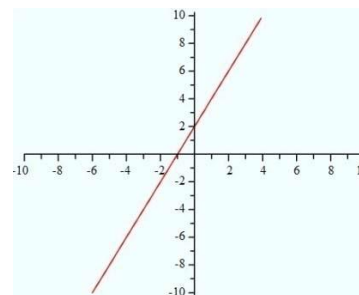
b)



c)



d)



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4. The function  $y = 3x - 5$ , which statement best describes the effect of increasing the y-intercept by 7?

- a. The new line is parallel to the original.
- b. The new line has greater rate of change.

5. Which statement best describes the effect on the graph of  $f(x) = 8x - 3$

If the y intercept is changed to + 5?

- a. The slope remains constant.
- b. The new line passes through the origin.

6. The function  $y = 4x - 6$ , which statement best describes the effect of increasing the y-intercept by 7?

- a. The new line is parallel to the original.
- b. The new line has greater rate of change.

7. Which statement best describes the effect on the graph of  $f(x) = 14x - 4$

If the y intercept is changed to + 6?

- a. The slope stays the same.
- b. The new line passes through the origin.

8. Which statement best describes the effect on the graph of  $f(x) = 5x - 8$

If the y intercept is changed to + 2?

- a. The slope does not change.
- b. The new line passes through the origin.

9. The function  $y = 10x - 12$ , which statement best describes the effect of increasing the y-intercept by 4?

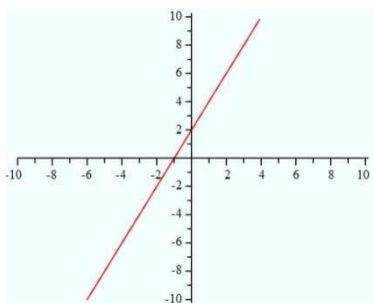
- a. The new line is parallel to the original.
- b. The new line has greater rate of change.

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Worksheet 39.3 Manipulating the Graphs of Functions **Answers**

**Manipulating the Graphs of Functions - Independent Practice  
Worksheet Answer Key**

1. a. Yes, the new line is parallel to the original.
2. a. Both lines are parallel, so the slope is the same.
3. d.



4. a. Yes, the new line is parallel to the original.
5. a. Both lines are parallel, so the slope is the same.
6. a. Yes, the new line is parallel to the original.
7. a. Both lines are parallel, so the slope is the same.
8. a. Both lines are parallel, so the slope is the same.
9. a. Yes, the new line is parallel to the original.

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**Application Activity: Comparing iPhone Plans**

**Which Plan Should You Get For Your iPhone?**

You can get the iPhone 6 for \$200 with a two-year contract on any of the following plans from Verizon, AT&T or Sprint.

Verizon	
Data	Monthly Cost
250 MB	\$55
500 MB	\$70
1 GB	\$80
2 GB	\$90
3 GB	\$100
4 GB	\$110
6 GB	\$120
8 GB	\$130
10 GB	\$140

AT&T	
Data	Monthly Cost
300 MB	\$60
1 GB	\$85
2 GB	\$95
4 GB	\$110
6 GB	\$120
8 GB	\$130
10 GB	\$140

Sprint	
Data	Monthly Cost
1 GB	\$70
Unlimited	\$80

Or with T-Mobile you can get a \$648 phone and pay the following monthly fees without a contract:

Data	Monthly Cost
1 GB	\$50
3 GB	\$60
5 GB	\$70
Unlimited	\$80

1. Pick a plan for your iPhone 6. Find your cost for the phone and service over time. Be sure to find the cost for the first several months, as well as after one, two and three years. You might find it useful to organize the costs over time in the table below (we have started it for you).

Months	0	1	2									
Cost												

2. What math did you do to find your total cost after 12, 24 and 36 months? What math are you doing over and over again regardless of the number of months?

3. Write a rule that gives your cost for the phone after any number of months. Be sure to describe what each variable or number in your equation represents in this context.

4. Graph the cost of your phone over three years. Consider which variable belongs on which axis and how to set up the scales for each axis.



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5. Where do you see the monthly rate of your cell phone plan in your graph? Where do you see the cost of your phone?

### **Should you pay full price for the phone with T – Mobile?**

6. Compare the 3 GB plan with Verizon for \$100 per month with the 3 GB plan from T–Mobile for \$60 per month by graphing each plan on the grid below. Over time which is a better deal? Should you pay full price for the phone and get the lower monthly rate with T–Mobile? Or should you get the phone for \$200 and pay the higher monthly rate with Verizon? For what number of months is T–Mobile cheaper? At what number of months is Verizon cheaper?

7. Which plan do you think is the best deal? What other factor are involved in determining the best deal and which carrier to use?

Source: <http://time.com/7982/which-wireless-plan-is-cheapest/>

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### Application Activity: Comparing iPhone Plans **Answers**

#### Which Plan Should You Get For Your iPhone?

You can get the iPhone 6 for \$200 with a two-year contract on any of the following plans from Verizon, AT&T or Sprint.

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Data	Monthly Cost
250 MB	\$55
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8 GB	\$130
10 GB	\$140

AT&T	
Data	Monthly Cost
300 MB	\$60
1 GB	\$85
2 GB	\$95
4 GB	\$110
6 GB	\$120
8 GB	\$130
10 GB	\$140

Sprint	
Data	Monthly Cost
1 GB	\$70
Unlimited	\$80

Or with T-Mobile you can get a \$648 phone and pay the following monthly fees without a contract:

Data	Monthly Cost
1 GB	\$50
3 GB	\$60
5 GB	\$70
Unlimited	\$80

1. Pick a plan for your iPhone 6. Find your cost for the phone and service over time. Be sure to find the cost for the first several months, as well as after one, two and three years. You might find it useful to organize the costs over time in the table below (we have started it for you).

Months	0	1	2	3	4	5	6	7	8	12	24	36
Cost												

Student's answers will vary depending on which plan they select. You might have kids use the x values that we've given in the table.

2. What math did you do to find your total cost after 12, 24 and 36 months? What math are you doing over and over again regardless of the number of months?

Consider having kids jump up to 12, 24, 36 or even 48 months. Break away from simply repeatedly adding the monthly fee. Have them compute for various months that are not consecutive, such as 12, 18, 24, 36 or 48. Have them focus on the math they are doing regardless of the number of months ... Multiply the months by the monthly fee and add in the cost of the phone.

3. Write a rule that gives your cost for the phone after any number of months. Be sure to describe what each variable or number in your equation represents in this context.

Answers will vary based on the plan they select. In the example equation  $y = 70x + 648$ , the y represents the cost, the 70 is the monthly fee, the x is the number of months and the 648 is the initial cost of the iPhone.

4. Graph the cost of your phone over three years. Consider which variable belongs on which axis and how to set up the scales for each axis.

Again answers will vary. Check out our sliders in the Desmos graphing calculator for this lesson. The link is in the post for this activity.

5. Where do you see the monthly rate of your cell phone plan in your graph? Where do you see the cost of your phone?

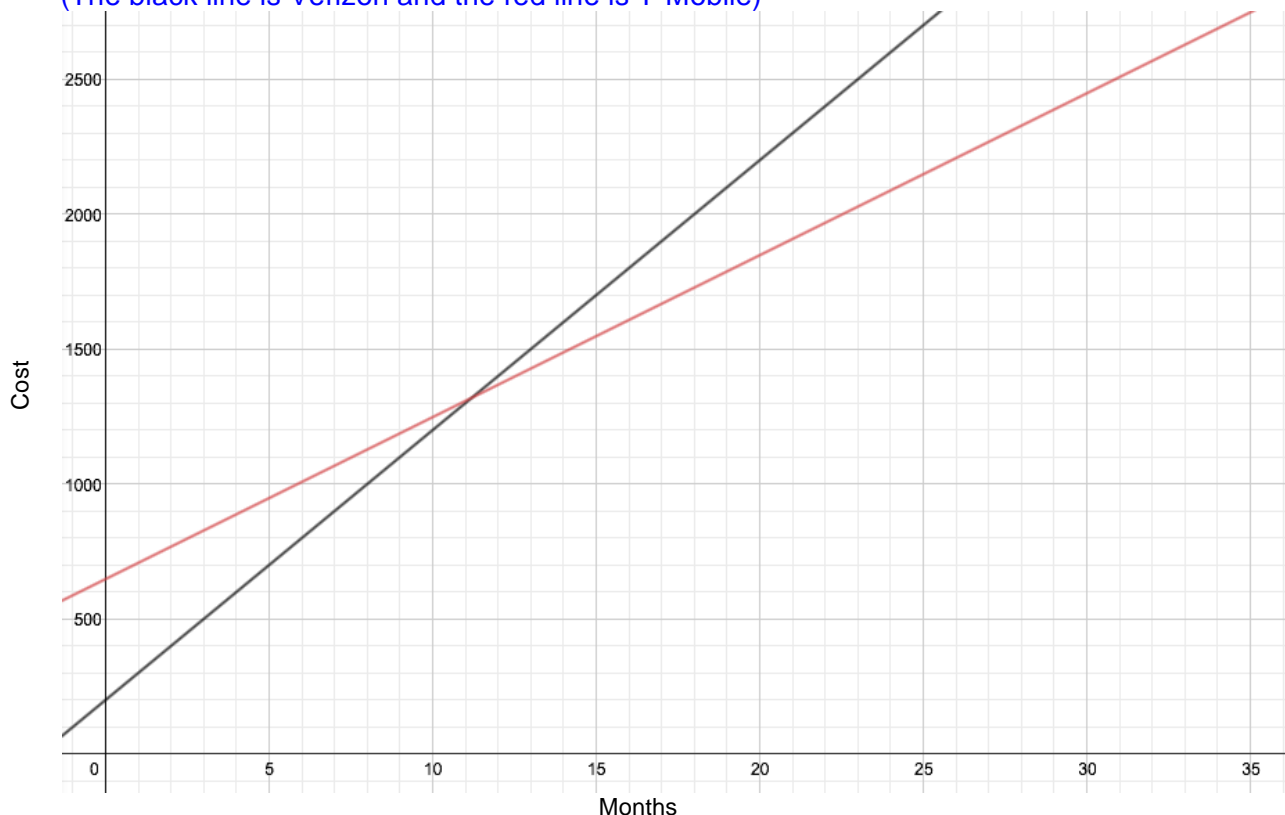
The cost of the phone is the y-intercept. The monthly fee is the slope.

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### Should you pay full price for the phone with T – Mobile?

6. Compare the 3 GB plan with Verizon for \$100 per month with the 3 GB plan from T – Mobile for \$60 per month by graphing each plan on the grid below. Over time which is a better deal? Should you pay full price for the phone and get the lower monthly rate with T – Mobile? Or should you get the phone for \$200 and pay the higher monthly rate with Verizon? For what number of months is T – Mobile cheaper? At what number of months is Verizon cheaper?

See the graph on the next page. Verizon is cheaper from months 0 – 12. They are the same at month 12. From month 13 and on, T-Mobile is the better deal.  
(The black line is Verizon and the red line is T-Mobile)



7. Which plan do you think is the best deal? What other factor are involved in determining the best deal and which carrier to use?

Students might notice the T-Mobile is considerably cheaper and going off cost, is the much better deal. With T-Mobile you save over \$500 over two years. Quality of cell phone service, coverage area and access/cost of features will also come into play.

Source: <http://time.com/7982/which-wireless-plan-is-cheapest/>

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