Weekly Focus: rational expressions
Weekly Skill: computation

LESSON 28: Rational Expressions and Equations part 1

Lesson Summary: For the warm-up, students will solve a problem about a patio. In Activity 1, they will simplify rational expressions. In Activity 2, students will add and subtract rational expressions, and in Activity 3, they will multiply and divide them. There is an exit ticket at the end. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 28:

- Video (length 6:37) on simplifying expressions
- Video (length 3:11) on adding and subtracting rational expressions. The videos are required for teachers and optional for students.
- 3 Notes sheets (28A, 28B, 28C) to be used for students and/or for teaching notes. These come from algebra2go.
- 4 Worksheets (28.1, 28.2, 28.3, 28.4) with answers (attached)

Objectives: Students will be able to:

- Solve the patio word problem with a quadratic equation
- Simplify rational expressions
- Add, subtract, multiply and divide rational expressions

ACES Skills Addressed: N, CT, LS
CCRS Mathematical Practices Addressed: Reason Abstractly and Quantitatively, Use Appropriate Tools Strategically
Levels of Knowing Math Addressed: Intuitive, Abstract

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 28 Warm-up: Solve the patio problem	Time: 5-10 Minutes

Write on the board: Felicia wants to increase the size of her outdoor patio by adding 2 feet to each side to make it 49 ft^2.

Basic Questions:

- What is the shape of her patio?
  - It is square because she added the same length to both sides and 7 x 7 = 49.
- What was the area of her patio before?
  - It was 25 square feet because it was 2 feet shorter on each side. It was 5 x 5 = 25.

Extension Questions:

- Solve the problem with expressions, multiplying, and factoring.
- Let x = length of one side of original patio
  - $(x + 2)(x + 2) = 49$
  - $x^2 + 4x + 4 = 49$
  - $x^2 + 4x - 45 = 0$
  - $(x - 5)(x + 9) = 0$
  - x = 5 or x = -9. Since length can't be negative, the original length of one side was 5 ft.
  - Check: (5 + 2) (5+ 2) = (7)(7) = 49. Correct.

Lesson 28 Activity 1: Simplifying Rational Expressions	Time: 20-25 Minutes

1. **Rational expressions** are like rational numbers (can't have a zero denominator) except that they have variables in the numerator and/or denominator.
2. Some examples are: $\frac{5}{x}$ and $\frac{n+1}{n-1}$ and $\frac{r^2+5r}{r^2}$
3. Rational expressions are in their simplest form if the numerator and denominator cannot be simplified further just like with regular fractions. The last example above can be simplified to $\frac{r+5}{r}$.
4. Examples:

1. $\frac{18}{60}$ can reduce by dividing both numerator and denominator by 6 to get 3/10

2. $\frac{7x-21}{28}$ can be reduced by taking 7 out of every term to get $\frac{x-3}{4}$

3. $\frac{30a^4b^2}{40a^2b^5}$ can be reduced by dividing both numerator and denominator by $10a^2b^2$ to get $\frac{3a^2}{4b^3}$

$\frac{3s-12t}{(s-4t)^2}$ can be reduced by diving both by (s – 4t) to get $\frac{3}{(s-4t)}$

Give students **Notes 28A** for more examples and explanations.
Practice with **Worksheet 28.1**. Have volunteers do some answers on the board.

Lesson 28 Activity 2: Adding and Subtracting Rational Expressions	Time: 30-35 Minutes

1. Like regular fractions, rational expressions can be added and subtracted once they have a common denominator.
2. Do the examples on **Notes 28B Adding and Subtracting Rational Expressions.** You can copy the notes for your students or have them take notes as you explain. (Note: LCD=lowest common denominator).
3. Do **Worksheet 28.2**
4. Have volunteers do a few problems on the board.

Lesson 28 Activity 3: Multiplying and Dividing Rational Expressions	Time: 35-40 Minutes

1. Like regular fractions, rational expressions can be multiplied or divided without needing a common denominator.
2. Do the examples on **Notes 28C Multiplying and Dividing Rational Expressions.** You can copy the notes for your students or have them take notes as you explain.
3. Do **Worksheet 28.3 Multiplying Rational Expressions**
4. Have volunteers do a few problems on the board.
5. Do **Worksheet 28.4 Dividing Rational Expressions**
6. Have volunteers do a few problems on the board.

Lesson 28 Exit Ticket: 2 true, 1 false	Time: 5 Minutes
Which of the following is not correct? 1. $\frac{5x}{5x+10} = \frac{x}{x+2}$ 2. $\frac{4x^2}{x} = 4x$ 3. $\frac{49xy}{7y} = 7xy$ *Answer: #3 is false, answer is 7x*	

Notes 28A Simplifying Expressions

Simplifying rational expressions

Consider the expression; Can we reduce?

$$\frac{2x+4}{2}$$

We CANNOT cancel the 2's, but we can factor the numerator

$$2x+4 = 2(x+2)$$

So our expression can be written

$$\frac{2(x+2)}{2}$$

Now the 2 in the numerator can be cancelled with the 2 in the denominator.

$$\frac{\cancel{2}(x+2)}{\cancel{2}} = \boxed{x+2}$$

Example 1: Simplify each expression:

A) $\frac{3x+12}{2x+8}$

We must factor the top and bottom first to see if any factors will cancel:

$$\frac{3x+12}{2x+8} = \frac{3(x+4)}{2(x+4)} = \boxed{\frac{3}{2}}$$ [the binomials cancel!]

B) $\frac{(x+3)(x+5)}{(x-3)(x+5)}$

Here the numerator and denominator are already factored. Since there is an $(x+5)$ in TOP AND in the BOTTOM, they will cancel.

$$\frac{(x+3)\cancel{(x+5)}}{(x-3)\cancel{(x+5)}} = \boxed{\frac{x+3}{x-3}}$$

c) $\frac{4x^2-4x}{5x-5}$

We need to factor the top and bottom first to see if any factors will cancel:

$$\frac{4x^2-4x}{5x-5} = \frac{4x(x-1)}{5(x-1)} = \boxed{\frac{4x}{5}}$$

D) $\frac{x^2+2x-15}{x^2+6x+5}$

We need to factor the top and bottom first to see if any factors will cancel.

Let's start with the top:

$x^2+2x-15$

We need to use the abc method.

$x^2 + 2x - 15$

$a = 1$

$b = 2$

$c = -15$

$a \cdot c = -15$

we need factors of -15 that add to 2 (the b term)

-15		sum
1	-15	-14
3	-5	-2
-3	5	2

so we get

$x^2 + 2x - 15 = (x-3)(x+5)$

Now the denominator:

$x^2 + 6x + 5$

$a = 1$

$b = 6$

$c = 5$

$a \cdot c = 5$

We need factors of 5 that add to 6 (the b term)

The factors are 5 and 1.

So we get

$$x^2+6x+5 = (x+5)(x+1)$$

Therefore our expression can be written:

$$\frac{x^2+2x-15}{x^2+6x+5} = \frac{(x-3)(x+5)}{(x+5)(x+1)}$$

Will anything cancel?

What's the final solution?

Answer: $\boxed{}$

E) $\frac{(6-x)}{(x-6)}$

We are hoping to factor the top or bottom so that something will cancel.

If we factor out a negative 1 from the top we get

$$\frac{6-x}{x-6} = \frac{-1(-6+x)}{x-6}$$

Using the commutative property of addition, we can write the top binomial as $x+(-6) = x-6$

So our expression becomes

$$\frac{-1(x-6)}{x-6}$$

The binomials cancel and our final answer is $\boxed{-1}$.

Worksheet 28.1 Simplifying Expressions

Simplify each expression.

1) $\dfrac{q^2 + 10q + 21}{q^2 + 7q + 12}$

6) $\dfrac{3n + 12}{21}$

2) $\dfrac{h - 4}{8h - 32}$

7) $\dfrac{12x^6}{16x}$

3) $\dfrac{12}{6p + 30}$

8) $\dfrac{5r - 5}{10}$

4) $\dfrac{72b^3}{45b^5}$

9) $\dfrac{54k^4}{27k^4}$

5) $\dfrac{18s^4}{12s^4}$

10) $\dfrac{z^2 - 14z + 45}{z^2 - z - 20}$

Worksheet 28.1 Answers

Simplify each expression.

1) $\dfrac{q^2 + 10q + 21}{q^2 + 7q + 12}$

$\dfrac{q + 7}{q + 4}$

6) $\dfrac{3n + 12}{21}$

$\dfrac{n + 4}{7}$

2) $\dfrac{h - 4}{8h - 32}$

$\dfrac{1}{8}$

7) $\dfrac{12x^6}{16x}$

$\dfrac{3x^5}{4}$

3) $\dfrac{12}{6p + 30}$

$\dfrac{2}{p + 5}$

8) $\dfrac{5r - 5}{10}$

$\dfrac{r - 1}{2}$

4) $\dfrac{72b^3}{45b^5}$

$\dfrac{8}{5b^2}$

9) $\dfrac{54k^4}{27k^4}$

2

5) $\dfrac{18s^4}{12s^4}$

$\dfrac{3}{2}$

10) $\dfrac{z^2 - 14z + 45}{z^2 - z - 20}$

$\dfrac{z - 9}{z + 4}$

Notes 28B Adding and Subtracting Rational Expressions

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Example 1: $\frac{4}{x} + \frac{1}{2x^2}$

Since these are NOT like terms, we must make them like terms.

We need to find the LCD.

$LCD = 2x^2$

The first fraction needs to be multiplied by $\frac{2x}{2x}$ to make the denominator $2x^2$. [the second fraction does not need to be changed]

$$\frac{4}{x}\left(\frac{2x}{2x}\right) + \frac{1}{2x^2} = \frac{8x}{2x^2} + \frac{1}{2x^2}$$

$$= \boxed{\frac{8x+1}{2x^2}}$$

Example 2: $\frac{3}{a} + \frac{2}{b}$

$LCD = ab$

$$\frac{3}{a}\left(\frac{b}{b}\right) + \frac{2}{b}\left(\frac{a}{a}\right)$$

$$= \frac{3b}{ab} + \frac{2a}{ab} = \boxed{\frac{3b+2a}{ab}}$$

example 3: $\frac{3}{x^2y} + \frac{2}{xy^2} - \frac{1}{xy}$

LCD $= x^2y^2$

$$\left(\frac{y}{y}\right)\frac{3}{x^2y} + \left(\frac{x}{x}\right)\frac{2}{xy^2} - \left(\frac{xy}{xy}\right)\frac{1}{xy}$$

$$= \frac{3y}{x^2y^2} + \frac{2x}{x^2y^2} - \frac{xy}{x^2y^2}$$

$$= \boxed{\frac{3y + 2x - xy}{x^2y^2}}$$

example 4: $\frac{1}{x+4} + \frac{2}{x}$

LCD $= x(x+4)$

$$\left(\frac{x}{x}\right)\frac{1}{x+4} + \left(\frac{x+4}{x+4}\right)\frac{2}{x}$$

$$= \frac{x}{x(x+4)} + \frac{2(x+4)}{x(x+4)}$$

$$= \frac{x + 2(x+4)}{x(x+4)}$$

Now simplify the numerator completely.

Note: There's no need to multiply the denominator out.

$$= \frac{x + 2x + 8}{x(x+4)} = \boxed{\frac{3x+8}{x(x+4)}}$$

Example 5: $\frac{x}{x-2} - \frac{8}{x^2-4}$

To find the LCD, we must first factor each denominator completely.

$$\frac{x}{x-2} - \frac{8}{(x+2)(x-2)}$$

LCD = $(x-2)(x+2)$

$$\left(\frac{x+2}{x+2}\right)\frac{x}{(x-2)} - \frac{8}{(x+2)(x-2)}$$

$$= \frac{x(x+2)}{(x+2)(x-2)} - \frac{8}{(x+2)(x-2)}$$

$$= \frac{x(x+2)-8}{(x+2)(x-2)}$$

now simplify the numerator completely

$$= \frac{x^2+2x-8}{(x+2)(x-2)}$$

We need to factor the numerator (if possible) to see if anything cancels.

$$= \frac{(x+4)(x-2)}{(x+2)(x-2)}$$

The $(x-2)$'s cancels so our final answer is $\boxed{\frac{x+4}{x+2}}$

Worksheet 28.2 Adding and Subtracting Rational Expressions

Add or Subtract the two expressions in each problem.

1) $\frac{7d}{d + 6} - \frac{6}{d + 3}$

6) $\frac{7y - 7n}{5y^5} - \frac{4y - 3n}{5y^5}$

2) $\frac{6p^2 + 3x^2}{5p^4} - \frac{2p^2 + 4x^2}{5p^4}$

7) $\frac{6n + 8y}{2n^2y^2} - \frac{6n - 9y}{2n^2y^2}$

3) $\frac{7h^3 - 8}{5h^5 - 2} - \frac{8h}{5h^5 - 2}$

8) $\frac{8c}{3c - 4} - \frac{2}{5c - 7}$

4) $\frac{5s^2 - 6}{8s^4 - 7} - \frac{3s}{8s^4 - 7}$

9) $\frac{7r}{4} - \frac{2r + 2}{3r + 6}$

5) $\frac{5k}{7} - \frac{6k + 6}{3k + 4}$

10) $\frac{8b - 4}{7b^4 - 5b} - \frac{2b - 3}{7b^4 - 5b}$

Worksheet 28.2 Answers

Add or Subtract the two expressions in each problem.

1) $\frac{7d}{d+6} - \frac{6}{d+3}$

$\frac{7d^2 + 15d - 36}{(d+6)(d+3)}$

2) $\frac{6p^2 + 3x^2}{5p^4} - \frac{2p^2 + 4x^2}{5p^4}$

$\frac{4p^2 - x^2}{5x^4}$

3) $\frac{7h^3 - 8}{5h^5 - 2} - \frac{8h}{5h^5 - 2}$

$\frac{-h^3 - 8}{5h^5 - 2}$

4) $\frac{5s^2 - 6}{8s^4 - 7} - \frac{3s}{8s^4 - 7}$

$\frac{2s^2 - 6}{8s^4 - 7}$

5) $\frac{5k}{7} - \frac{6k + 6}{3k + 4}$

$\frac{15k^2 - 22k - 42}{7(3k + 4)}$

6) $\frac{7y - 7n}{5y^5} - \frac{4y - 3n}{5y^5}$

$\frac{3y - 4n}{5n^5}$

7) $\frac{6n + 8y}{2n^2y^2} - \frac{6n - 9y}{2n^2y^2}$

$\frac{17}{2y}$

8) $\frac{8c}{3c - 4} - \frac{2}{5c - 7}$

$\frac{2(20c^2 - 31c + 4)}{(3c - 4)(5c - 7)}$

9) $\frac{7r}{4} - \frac{2r + 2}{3r + 6}$

$\frac{21r^2 + 34r - 8}{12(r + 2)}$

10) $\frac{8b - 4}{7b^4 - 5b} - \frac{2b - 3}{7b^4 - 5b}$

$\frac{6b - 1}{7b^4 - 5b}$

Notes 28C Multiplying and Dividing Rational Expressions

multiplying and dividing rational expressions

Recall: $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \cdot \frac{8}{7}$

$\frac{3}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{7} = \frac{3 \cdot 2}{1 \cdot 7} = \boxed{\frac{6}{7}}$

example 1:

A) $\frac{x}{4} \div \frac{7}{8}$

$\frac{x}{4} \div \frac{7}{8} = \frac{x}{4} \cdot \frac{8}{7}$

$\frac{x}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{7} = \boxed{\frac{2x}{7}}$

B) $\frac{x+3}{x-2} \div \frac{x-1}{x-2}$

$\frac{x+3}{x-2} \div \frac{x-1}{x-2} = \frac{x+3}{x-2} \cdot \frac{x-2}{x-1}$

$\frac{(x+3)}{_1\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}^1}{(x-1)} = \boxed{\frac{x+3}{x-1}}$

C) $\frac{(x+2)^2}{x+1} \cdot \frac{x-1}{x+2}$

Here the binomial x+2 in the bottom will cancel with one of the x+2's in the top.

If we write it out we get

$$\frac{(x+2)^2}{x+1}\cdot\frac{x-1}{x+2}=\frac{(x+2)(x+2)}{x+1}\cdot\frac{x-1}{x+2}$$

$$\frac{(x+2)\cancel{(x+2)}}{x+1}\cdot\frac{x-1}{\cancel{x+2}}=\boxed{\frac{(x+2)(x-1)}{(x+1)}}$$

Steps for multiplying and dividing rational expressions:

Step 1: Make all division problems into multiplication problems (see example 1-A)

Step 2: Factor the numerator and denominator completely.

Step 3: Cancel out all factors that are in the numerator AND the denominator (reduce)

Note: Keep your final answer in factored form – you do not need to multiply or distribute terms.

example 2:

$$\frac{x^2+6x+9}{x^2-4} \cdot \frac{x-2}{x+3}$$

step 1: This step is not needed since it is already a multiplication problem.

step 2: Factor.

$$\frac{x^2+6x+9}{x^2-4} \cdot \frac{x-2}{x+3} = \frac{(x+3)(x+3)}{(x+2)(x-2)} \cdot \frac{(x-2)}{(x+3)}$$

step 3: cancel factors that appear on top AND bottom.

$$\frac{\cancel{(x+3)}(x+3)}{(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}}{\cancel{(x+3)}}$$

so our final answer is $\boxed{\dfrac{x+3}{x+2}}$

example 3:

$$\frac{x^2+6x+8}{x^2-x-6} \div \frac{x^2+3x-4}{x^2+2x-3}$$

step 1: $\dfrac{x^2+6x+8}{x^2-x-6} \cdot \dfrac{x^2+2x-3}{x^2+3x-4}$

Step 2: Factor

$$\frac{x^2+6x+8}{x^2-x-6} \cdot \frac{x^2+2x-3}{x^2+3x-4} = \frac{(x+2)(x+4)}{(x-3)(x+2)} \cdot \frac{(x+3)(x-1)}{(x+4)(x-1)}$$

Step 3: Cancel factors that appear on top AND bottom.

$$\frac{\cancel{(x+2)}\cancel{(x+4)}}{(x-3)\cancel{(x+2)}} \cdot \frac{(x+3)\cancel{(x-1)}}{\cancel{(x+4)}\cancel{(x-1)}}$$

So our final answer is

$$\boxed{\frac{x+3}{x-3}}$$

Finding the LCD of Rational Expressions Handout

Steps to finding the LCD of rational expressions:

Step 1: Factor each denominator completely

Step 2: List each different factor from step 1, the greatest number of times it appears in a denominator.

Step 3: Multiply the factors from step 2 — this is the LCD

Example: Find the LCD of the terms

$$\frac{4}{x}, \frac{3}{x+1}, \frac{2}{x^2+2x+1}, \frac{1}{x^2+3x+2}$$

There are four denominators:

$x, x+1, x^2+2x+1, x^2+3x+2$

Step 1: Factoring each denominator we get

$x, x+1, (x+1)^2, (x+1)(x+2)$

Step 2: The different factors are

x, $x+1$ and $x+2$

the highest power of x is 1,

the highest power of $(x+1)$ is 2

and the highest power of $(x+2)$ is 1

Step 3: LCD $= x(x+1)^2(x+2)$

Worksheet 28.3 Multiplying Rational Expressions

Simplify each expression.

1) $\frac{8}{4} \cdot \frac{6}{10c}$

2) $\frac{(d + 12)(d - 3)}{d + 12} \cdot \frac{2}{(d + 9)(d + 12)}$

3) $\frac{8}{(h + 11)} \cdot \frac{4h - 48}{(h - 12)}$

4) $\frac{45z^2 + 45z}{18z^2 + 18z} \cdot \frac{8z}{8}$

5) $\frac{9}{8} \cdot \frac{10}{12x}$

6) $\frac{45y^2 - 45y}{55y^2 - 55y} \cdot \frac{10y}{10}$

7) $\frac{8(q - 6)}{6} \cdot \frac{4q}{8(q - 6)}$

8) $\frac{(p - 8)(p - 5)}{p - 8} \cdot \frac{12}{(p + 2)(p - 8)}$

9) $\frac{2(r - 4)}{(r - 4)} \cdot \frac{5r}{2(r + 6)}$

10) $\frac{12(b - 6)}{6} \cdot \frac{8b}{12(b - 6)}$

Worksheet 28.3 Answers

Simplify each expression.

1) $\frac{8}{4} \cdot \frac{6}{10c}$

$\frac{6}{5c}$

2) $\frac{(d + 12)(d - 3)}{d + 12} \cdot \frac{2}{(d + 9)(d + 12)}$

$\frac{2(d - 3)}{(d + 9)(d + 12)}$

3) $\frac{8}{(h + 11)} \cdot \frac{4h - 48}{(h - 12)}$

$\frac{32}{h + 11}$

4) $\frac{45z^2 + 45z}{18z^2 + 18z} \cdot \frac{8z}{8}$

$\frac{5z}{2}$

5) $\frac{9}{8} \cdot \frac{10}{12x}$

$\frac{15}{16x}$

6) $\frac{45y^2 - 45y}{55y^2 - 55y} \cdot \frac{10y}{10}$

$\frac{9y}{11}$

7) $\frac{8(q - 6)}{6} \cdot \frac{4q}{8(q - 6)}$

$\frac{2q}{3}$

8) $\frac{(p - 8)(p - 5)}{p - 8} \cdot \frac{12}{(p + 2)(p - 8)}$

$\frac{12(p - 5)}{(p + 2)(p - 8)}$

9) $\frac{2(r - 4)}{(r - 4)} \cdot \frac{5r}{2(r + 6)}$

$\frac{5r}{r + 6}$

10) $\frac{12(b - 6)}{6} \cdot \frac{8b}{12(b - 6)}$

$\frac{4b}{3}$

Worksheet 28.4 Dividing Rational Expressions

Simplify each expression.

1) $\frac{11p}{2} \div \frac{7}{4}$

2) $\frac{33y^2 + 19y - 60}{27y^2 + 75y + 50} \div \frac{y^2}{36y^2 + 112y + 80}$

3) $\frac{10k^2}{3} \div \frac{9k}{8}$

4) $\frac{84b - 77}{8} \div \frac{108b - 99}{8b}$

5) $\frac{q^2 + 12q + 27}{q^2 + 14q + 33} \div \frac{1}{q + 11}$

6) $\frac{5x}{x + 4} \div \frac{5x}{7x + 28}$

7) $\frac{d + 7}{d + 12d + 35} \div \frac{6d}{d + 9}$

8) $\frac{6g}{g + 5} \div \frac{6g}{10g + 50}$

9) $\frac{5r}{10} \div \frac{12}{11}$

10) $\frac{6n}{11} \div \frac{5}{4}$

Worksheet 28.4 Answers

Simplify each expression.

1) $\frac{11p}{2} \div \frac{7}{4}$

$\frac{22p}{7}$

2) $\frac{33y^2 + 19y - 60}{27y^2 + 75y + 50} \div \frac{y^2}{36y^2 + 112y + 80}$

$\frac{4(11y - 12)(1y + 2)}{y^2}$

3) $\frac{10k^2}{3} \div \frac{9k}{8}$

$\frac{80k}{27}$

4) $\frac{84b - 77}{8} \div \frac{108b - 99}{8b}$

$\frac{7b}{9}$

5) $\frac{q^2 + 12q + 27}{q^2 + 14q + 33} \div \frac{1}{q + 11}$

$q + 9$

6) $\frac{5x}{x + 4} \div \frac{5x}{7x + 28}$

7

7) $\frac{d + 7}{d + 12d + 35} \div \frac{6d}{d + 9}$

$\frac{d + 9}{6d(d + 5)}$

8) $\frac{6g}{g + 5} \div \frac{6g}{10g + 50}$

10

9) $\frac{5r}{10} \div \frac{12}{11}$

$\frac{11r}{24}$

10) $\frac{6n}{11} \div \frac{5}{4}$

$\frac{24n}{55}$