Weekly Focus: quadratic equations
Weekly Skill: factoring

LESSON 26: Quadratic Equations part 1

Lesson Summary: For the warm-up, students will solve a problem about oil usage. Activity 1 is an introduction to quadratic equations and polynomials vocabulary. Activities 2 and 3 are multiplying polynomials. Activity 4 is factoring quadratic equations. There is also an exit ticket. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 26:

- Video A (length 6:46) on multiplying binomials with the FOIL method
- Video B (length 10:00) on factoring. The videos are required for teachers and recommended for students.
- 1 Notes (26.1) handout (attached)
- 3 Worksheets (26.2, 26.3, 26.4) with answers (attached)
- Note: Websites for more practice: kutasoftware and math-aids.

Objectives: Students will be able to:

- Understand the difference between linear equations and quadratic equations
- Multiply polynomials
- Factor quadratic equations

ACES Skills Addressed: N, CT, LS, ALS
CCRS Mathematical Practices Addressed: Make Sense of Problems and Persevere in Solving Them, Mathematical Fluency
Levels of Knowing Math Addressed: Intuitive, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 26 Warm-up: Solve the oil problem	Time: 5 Minutes

Write on the board: The Garcia family owns a home heated by oil and their furnace uses an average of 90 gallons per month. A new programmable thermostat could cut their oil usage by 15%.

Basic Question:

- How many gallons of oil would be saved in 4 months?
  - 4 months x 15% of 90 gallons = 4 x 0.15 x 90 = 54 gallons

Extension Question:

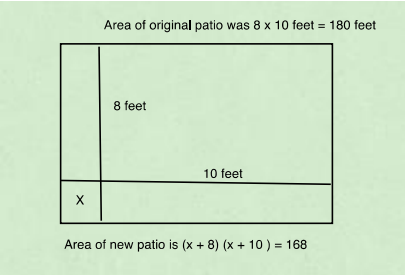
- Write an equation using *s* for the savings.
  - $s = 4\ (0.15)(90)$

Lesson 26 Activity 1: Intro to Quadratic Equations	Time: 10 Minutes

1. In previous lessons, we learned to solve linear equations with one or two variables. They are useful for solving many real-life problems.
2. Today we will learn about quadratic equations. They are also useful in solving real-life problems.
3. Quadratic equations differ from linear equations in that the variable can have an exponent of 2 and the graph is a curve, not a line. Graphing will be done in later lessons.

4. When do we use quadratic equations? Here is an example:

You have a patio that is 8 ft. by 10 ft. You want to increase the size of the patio to 168 square ft. by adding the same length to both sides of the patio. Let x = the length you will add to each side of the patio. You find the area of a rectangle by multiplying the length times the width. The new area of the patio will be 168 square ft.



a. When we find the new area, we have to multiply (x + 8)(x + 10) = 168
b. How do we do that? We multiply the expressions. We use the multiplication method called FOIL. We will practice that in Activity 3.
c. After that, we will solve the quadratic equation with factoring in Activity 4.

5. Give students the **Notes 26.1: Quadratic Equations Vocabulary and Factoring Notes.** Explain the vocabulary about polynomials.

6. Note to teacher: I tried to keep one example of a patio through activities 1, 3 and 4. If you have space on the board, keep it as a running example for each skill and to help make sense of when we use quadratic equations.

Lesson 26 Activity 2: Multiplying Polynomials	**Time: 20 Minutes**

1) The first type of multiplication you will do is multiplying monomials by polynomials.
2) Do the examples in the notes.
3) Practice with **Worksheet 26.2.** Do a few problems on the board.
4) Have volunteers solve a few others on the board.

Lesson 26 Activity 3: Multiplying Binomials	**Time: 35 Minutes**

1) The first type of multiplication you will do is multiplying binomials.
2) Let's use our example of the patio from earlier. We use what is called the **FOIL method (First, Outer, Inner, Last.**
3) Recall that the new area for the patio is (x + 8) (x + 10). Here is how to multiply:

(x + 8)(x + 10) = 168	
FOIL the factors (x + 8)(x + 10).	$x^2 + 10x + 8x + 80 = 168$
Simplify.	$x^2 + 18x + 80 = 168$
Subtract 168 from both sides of the equation.	$x^2 + 18x + 80 - 168 = 168 - 168$
Simplify both sides of the equation.	$x^2 + 18x - 88 = 0$

4) Now the equation is in a form that we can factor. Before we do that, let's practice multiplying binomials with the FOIL method.
5) **Do the examples** in the notes you have.
6) Practice with **Worksheet 26.3.** Do a few problems on the board.
7) Have volunteers solve a few others on the board.

Lesson 26 Activity 4: Factoring Quadratics	Time: 30 Minutes

1) After you multiply binomials, you have an equation in the **standard form of $ax^2 + bx + c = 0$** where **a** and **b** are coefficients and **c** is a constant.

2) Now you can solve the equation by factoring. Factoring means that you take the equation and divide it into 2 expressions with parentheses and then set each of those equal to zero. You will get two answers and one or both of those answers will make sense.

3) In factoring, you do the opposite of what you did in the FOIL method. The difference is when you factor, you have the equation with 0 on one side of the equal side so it is in a format that is possible for factoring. We would not have been able to solve the equation had we left it in its original form.

4) Let's continue with our patio example: $x^2 + 18x - 88 = 0$

Factor.	$(x + 22)(x - 4) = 0$
Set each factor equal to zero.	$x + 22 = 0$ and $x - 4 = 0$
Solve the equation.	$x + 22 = 0$
Subtract 22 from both sides of the equation.	$x + 22 - 22 = 0 - 22$
Simplify both sides of the equation.	$x = -22$
Solve the equation.	$x - 4 = 0$
Add 4 to both sides of the equation.	$x - 4 + 4 = 0 + 4$
Simplify both sides of the equation.	$x = 4$

Because this is a quadratic equation, you can expect two answers. The answers are 4 and –22. However, –22 is not a reasonable answer. You cannot have a negative length. Therefore, the only answer is 4.

To check your calculations, review the original dimensions of the patio—8 ft. by 10 ft. If you were to add 4 to each side, the new dimensions would be 12 ft. by 14 ft. When you multiply 12 times 14, you get 168 square ft., which is the new area you wanted.

5) Do **Worksheet 26.4** for practice. Do a few together first on the board.
6) Check your answers by multiplying the polynomials like you did in the first two worksheets.

Lesson 26 Exit Ticket	Time: 5 Minutes

1. Multiply x(x + 22)
2. Multiply (x + 3) (x – 4)
3. Factor $x^2 – 4x – 12$

Answers: $x^2 + 22x$, $x^2 – x – 12$, and $(x – 6)(x+2)$

Notes 26.1: Quadratic Equations Vocabulary and Factoring

In solving word problems with quadratic equations, we need to understand the vocabulary, how to multiply (simplify) terms, and how to factor the quadratic equations.

Definition of a quadratic equation:

A quadratic equation contains an x^2 term as well as an x term.

When it is graphed, it is not a straight line; it is a parabola, a smooth curve.

Vocabulary

Quadratic equations contain many terms. One term is called a **monomial.** Two terms are called **binomials**. Three terms are called **trinomials**. Terms are also referred to as **polynomials.**

A **polynomial** is a number, a variable, or a combination of a number and a variable. A polynomial can be one or more terms. Monomials, binomials, and trinomials are all polynomials.

Multiplying a Polynomial by a Monomial

A polynomial with one term is called a **monomial**. To multiply a polynomial with one term (monomial) by a polynomial with more than one term, use the distributive property. You multiply the term outside the parentheses by every term inside the parentheses.

Examples:

$2(a + b - 3) = 2a + 2b - 6$

$3x(x^2 + 2x) = 3x^3 + 6x^2$

Multiplying a Binomial by a Binomial

What is a binomial? A **binomial** is a polynomial with two terms. To multiply a binomial by a binomial, you will use a method called "FOIL." This process is called FOIL because you work the problem in this order:

FIRST

OUTER

INNER

LAST

Example: $(x + 2)(x + 3)$

Multiply the **first** terms in each binomial.	$([x] + 2)([x] + 3)$ $= x^2$
Multiply the two **outer** terms in each binomial.	$([x] + 2)(x + [3])$ $= x^2 + 3x$
Multiply the two **inner** terms in each binomial.	$(x + [2])([x] + 3)$ $= x^2 + 3x + 2x$
Multiply the two **last** terms in each binomial.	$(x + [2])(x + [3])$ $= x^2 + 3x + 2x + 6$
Simplify.	$= x^2 + 5x + 6$

Example: $(x + 3)(x - 1)$

Multiply the **first** terms in each binomial.	$([x] + 3)([x] - 1)$ $= x^2$
Multiply the two **outer** terms in each binomial.	$([x] + 3)(x - [1])$ $= x^2 - 1x$
Multiply the two **inner** terms in each binomial.	$(x + [3])([x] - 1)$ $= x^2 - 1x + 3x$
Multiply the two **last** terms in each binomial.	$(x + [3])(x - [1])$ $= x^2 - 1x + 3x - 3$
Simplify.	$= x^2 + 2x - 3$

Example: $(2x + 1)(3x - 2)$

Multiply the **first** terms in each binomial.	$([2x] + 1)([3x] - 2)$ $= 6x^2$
Multiply the two **outer** terms in each binomial.	$([2x] + 1)(3x - [2])$ $= 6x^2 - 4x$
Multiply the two **inner** terms in each binomial.	$(2x + [1])([3x] - 2)$ $= 6x^2 - 4x + 3x$

Multiply the two **last** terms in each binomial.	$(2x + [1])(3x - [2])$	
	$= 6x^2 - 4x + 3x - 2$	
Simplify.	$= 6x^2 - x - 2$	

Worksheet 26.2 Multiplying Monomials and Polynomials
Simplify each expression.

1) $6(3n + 9)$

2) $8(5s - 4)$

3) $4(9r^2 + 7r - 6)$

4) $3s(5s + 7d)$

5) $8g(9g + 6)$

6) $4(9p - 6n)$

7) $9(2q^2 + 4q - 6)$

8) $2k(7k - 9)$

9) $6n(5n + 9)$

10) $3(8k - 7c)$

Worksheet 26.2 Answers

1) 6(3n + 9)

18n + 54

2) 8(5s - 4)

40s - 32

3) $4(9r^2 + 7r - 6)$

$36r^2 + 28r - 24$

4) 3s(5s + 7d)

$15s^2 + 21sd$

5) 8g(9g + 6)

$72g^2 + 48g$

6) 4(9p - 6n)

36p - 24n

7) $9(2q^2 + 4q - 6)$

$18q^2 + 36q - 54$

8) 2k(7k - 9)

$14k^2 - 18k$

9) 6n(5n + 9)

$30n^2 + 54n$

10) 3(8k - 7c)

24k - 21c

Worksheet 26.3 Multiplying with the FOIL Method

Find the product of each expression.

1) $(g + 9)(g - 9)$

2) $(8s + 9)(8s - 9)$

3) $(g - 5)^2$

4) $(c + 4)^2$

5) $(8x - 4)^2$

6) $(4y + 7)^2$

7) $(g + 3)(g - 3)$

8) $(3d + 9)(3d - 9)$

9) $(y + 6)^2$

10) $(q + 5)(q - 5)$

Worksheet 26.3 Answers

Find the product of each expression.

1) $(g + 9)(g - 9)$

$g^2 - 81$

2) $(8s + 9)(8s - 9)$

$64s^2 - 81$

3) $(g - 5)^2$

$g^2 - 10g + 25$

4) $(c + 4)^2$

$c^2 + 8c + 16$

5) $(8x - 4)^2$

$64x^2 - 64x + 16$

6) $(4y + 7)^2$

$16y^2 + 56y + 49$

7) $(g + 3)(g - 3)$

$g^2 - 9$

8) $(3d + 9)(3d - 9)$

$9d^2 - 81$

9) $(y + 6)^2$

$y^2 + 12y + 36$

10) $(q + 5)(q - 5)$

$q^2 - 25$

Worksheet 26.4 Factoring Quadratics

1) $(h^2 - 9h)$

2) $(h^2 + 5h)$

3) $(q^2 - 9)$

4) $(n^2 - 9)$

5) $(g^2 - 4)$

6) $(q^2 - 6q)$

7) $g^2 - 7g + 12$

8) $m^2 + 14m + 45$

9) $z^2 + 11z + 28$

10) $m^2 + m - 20$

Worksheet 26.4 Answers

1) $(h^2 - 9h)$

$h(h - 9)$

2) $(h^2 + 5h)$

$h(h + 5)$

3) $(q^2 - 9)$

$(q - 3)(q + 3)$

4) $(n^2 - 9)$

$(n - 3)(n + 3)$

5) $(g^2 - 4)$

$(g - 2)(g + 2)$

6) $(q^2 - 6q)$

$q(q - 6)$

7) $g^2 - 7g + 12$

$(g - 4)(g - 3)$

8) $m^2 + 14m + 45$

$(m + 9)(m + 5)$

9) $z^2 + 11z + 28$

$(z + 4)(z + 7)$

10) $m^2 + m - 20$

$(m - 4)(m + 5)$